Aalto University
School of Business

WHY LOW-VOLATILITY INVESTING WORKS IN NORDIC MARKETS – DOES SIZE MATTER?

Master's Thesis
Pasi Koskela
Aalto University School of Business
Finance
Spring 2019

Aalto University
School of Business

Aalto University, P.O. BOX 11000, 00076 AALTO
www.aalto.fi

Abstract of master's thesis

Author Pasi Koskela	
Title of thesis Why low-volatility investing works in Nordic markets – does size matter?	
Degree Master of Science (Economics and Business Administration)	
Degree programme Finance	
Thesis advisor(s) Vesa Puttonen	
Year of approval 2019	**Number of pages** 77 **Language** English

Abstract

Stocks with past low idiosyncratic volatility deliver high future returns and significantly outperform stocks with high idiosyncratic volatility in the Nordic stock market over a sample period from January 2001 to December 2017. For the Nordic market, I show that the low-volatility anomaly exists with cross-sectional Fama Macbeth coefficient -1.26 and robust t-statistics -4.92. The effect is observed with equal-weighted returns in the aggregated Nordic market but also individually in Finland, Denmark, and Sweden. With value-weighted returns, the effect is significant and robust in all Nordic markets, including Norway. Size and quality, or other conventional controls, fail to explain IVOL thoroughly. Aggregated Nordic long-short IVOL portfolios among medium-sized stocks deliver a large, significant monthly FF-3 alpha of 1.6% with a 1.5% excess return. IVOL is the strongest amongst underpriced big and medium-sized stocks as well as portfolios with junk or neutral stocks. For the United States, IVOL remains controversially insignificant over the sample period from 2001 to 2017. As a reference, and consistent with past literature, an earlier sample period of 1980-2003 is also examined herein with significant coefficients for the United States.

Keywords Asset pricing, low volatility, IVOL, quality, size effect, Nordic market

Aalto-yliopisto
Kauppakorkeakoulu

Aalto-yliopisto, PL 11000, 00076 AALTO
www.aalto.fi

Maisterintutkinnon tutkielman tiivistelmä

Tekijä Pasi Koskela

Työn nimi Miksi matala volatiliteetti tuottaa Pohjoismaissa – vaikuttaako koko?

Tutkinto Kauppatieteiden maisteri

Koulutusohjelma Rahoitus

Työn ohjaaja(t) Vesa Puttonen

Hyväksymisvuosi 2019 **Sivumäärä** 77 **Kieli** Englanti

Tiivistelmä

Pohjoismaisella markkinalla matalan volatiliteetin osakkeet tuottavat tulevaisuudessa merkittävästi paremmin kuin osakkeet, joilla on korkea idiosynkraattinen volatiliteetti. Tutkielmani kattaa pohjoismaisen osakemarkkinan aikavälillä tammikuu 2001 – joulukuu 2017. Matalan volatiliteetin anomalia löytyy pohjoismaiselta markkinalta, sillä tuottojen poikkileikkauksessa Fama Macbeth-regression kerroin -1.26 on merkittävästi nollasta poikkeava robustilla t-arvolla -4.92. Ilmiö on havaittavissa tasapainoilla painotetuissa pohjoismaisen yhdistetyn markkinan tuotoissa, mutta myös erikseen Suomessa, Tanskassa ja Ruotsissa. Markkina-arvoilla tuottoja painotettaessa IVOL-efekti havaitaan yhdistetyssä pohjoismaisessa markkinassa, kaikilla markkinoilla erikseen, ja lisäksi myös Norjassa. Pohjoismaisen markkinan keskikokoisten yhtiöiden portfoliossa IVOL tuottaa keskimäärin kuukausittaista FF-3 alphaa 1.6% ylituoton ollessa 1.5%. Vahvimmillaan anomalia on aliarvostetuissa isoissa osakkeissa ja keskikokoisissa osakkeissa, joiden laatu on korkeimmillaan kahdessa alimmassa kolmanneksessa. Yhdysvalloissa anomaliaa ei yllättäen havaita tutkimukseni ajanjaksolla, joten analysoin vertailun vuoksi kirjallisuudessa aiemman esitetyn ajanjakson tuottoja tutkielmani mukaisilla menetelmillä. Anomalia löytyy aiemmin raportoitujen merkittävyyksien ja voimakkuuksien mukaisella tasolla Yhdysvaltojen osakkeiden tuotoista ajanjaksolle 1980-2003.

Avainsanat Omaisuuserien arvostus, matala volatiliteetti, IVOL, laatu, koko, Pohjoismaat

Table of contents

List of Tables

List of Figures

1 Introduction

High volatility should indicate high future return for stocks given that investors expect to earn a higher return from taking a higher risk. However, in the case of low-volatility anomaly, high volatility stocks fail to deliver high returns while the stocks with low volatility offer superior performance. Low-volatility anomaly challenges the fundamental finance theory of getting rewarded from greater risk. Higher-than-average risk should be compensated with higher-than-average returns. Researchers have shown on multiple occasions that high-volatility stocks offer low abnormal returns, which runs counter to the theory. Less volatile stocks seem to deliver higher risk-adjusted returns than stocks with higher volatility. In this study, low-volatility anomaly relates mostly to idiosyncratic stock-level volatility, described as IVOL, referring to the anomaly, also called a volatility puzzle. In addition to examining idiosyncratic volatility empirically in cross-sections and in time series, similar methods are applied to provide evidence of total volatility as a comparison.

The Nordic region is examined for the low-volatility anomaly as a joint market as well as for its constituents, Finland, Denmark, Norway, and Sweden, individually. Individual Nordic markets are relatively small and include a limited amount of large cap companies to be invested in and accessed by a global investor. However, by combining these individual markets, the aggregated Nordic market creates a meaningful base for a research attempt. The primary goal of the paper is to examine whether the anomaly exists in the Nordics, and if so, where particularly, among which kind of stocks, and whether it is robust after controlling for size and quality. I do not necessarily try to provide fully explained reasoning for why the anomaly is observed or why it exists in the first place. Prior literature offers several possible explanations for that, including some explanations which are possibly theoretically defendable, but the matter of causality could be questioned. The motivation for the study is the essential characteristics of the volatility as it is well known to remain persistent (e.g., Engle 1982). Despite that tendency, idiosyncratic volatility is shown to have time-varying behaviour, which indicates predictive power for future returns.

Although the anomaly is well researched within multiple markets, the Nordic aggregated market is hardly covered in prior literature. Some limited attempts exist but they are typically done using a pan-European, and not regional, Fama French (1993) three-factor model to extract idiosyncratic volatility for Nordic stocks. European factors are a possible source of bias, and therefore, in this study, I concentrate on the Nordic stock market using regionally computed FF-3, idiosyncratic volatility (IVOL), and quality (QMJ) factors. Stocks from Denmark, Finland, Norway, and Sweden are included. Iceland is excluded as the country has been exposed to the banking sector to a large extent and remained in the middle of the debt burden from the global crisis of 2007. The United States is included as a benchmark using equal sample period. To validate the methodology implementation, I extend the analysis to cover the U.S. sample period of 1980-2003, which is similar to the main reference paper of the topic. The U.S. is expressed in dollar terms while EUR-denominated numbers are used for the Nordic. With such an arrangement, the comparability to past literature is ensured while the underlying methodology is equally applied to analyse Nordics and its constituent markets.

The Fama Macbeth (1973) regression is applied as the primary method for examining the phenomenon on the individual stock level. Then, single- and double-sort portfolios are examined to show whether the effect is investable and for providing detailed information on performance and risk factor exposures. For the analysis, I compute all factors for the region to avoid possible biases from using ready-made European or International risk factors for explaining and validating local returns. The size effect has played a role in earlier research attempts as most of the papers report that the results deviate from each other by replacing equal-weighted weighting scheme with value-weighting for the portfolio returns. Therefore, OLS[1] and WLS[2] models, and multiple size-related sorts are applied for the portfolios to examine the role of the size comprehensively. I also examine the Nordic large cap separately to take into account the variation from illiquid small and micro-cap stocks.

The contribution of the paper includes three primary findings. First, the negative relationship between prior-month idiosyncratic volatility and next-month average returns across the market exists in the Nordic region. With equal-weighted returns, the phenomenon is observed in the aggregated Nordic market and individually in Finland, Denmark, and Sweden. With value-weighted returns, the anomaly is observed in all Nordic constituent markets, including Norway. Observed IVOL effect is examined by controlling for conventional firm and market-specific characteristics, especially against size. I report significant coefficients from the Fama Macbeth cross-section, and in addition, single-sorted portfolios - first with respect to IVOL, and then

[1] OLS, ordinary least squares linear regression assumes equal weight for each observation
[2] WLS, weighted least squares, generalisation of OLS, accounts for weight of each observation

double sorting for size delivers significant excess returns and alpha for the long-short portfolio. The portfolio is long, low-idiosyncratic volatility stocks and short, high-IVOL stocks.

The second finding relates to the United States, which is included in the paper as a reference market for the study. I report no significant evidence on low-volatility anomaly for the sample period in the United States. The finding is contradictory to earlier papers in which, for example, Ang et al. (2009) provide clear evidence of significant and robust low-volatility anomaly existence in the U.S. market. As a proof of methodology implementation, I examine a similar sample period with past literature and report significant results in the U.S. for the sample period of 1980-2003.

Finally, the last primary finding relates to the quality of the stock. Low-volatility anomaly in the Nordic aggregated market remains significant after the control for quality. The quality is measured as a QMJ score from Asness et al. (2014). The IVOL anomaly is strongest among medium-sized and large firms, but also among the junk and neutral stocks. Low-volatility survives the control for quality by showing significant but diminished, performance in self-financed long-short portfolios which are first sorted based on previous-period IVOL and then on beginning-of-month QMJ. For the long-only strategy, the most significant excess returns and alphas are reported on the portfolio with the lowest idiosyncratic volatility and the highest quality combined, which is new evidence. According to past research, IVOL typically succeeds in the short-leg with small and illiquid stocks and fails in the long-leg with larger and more liquid stocks. Furthermore, the findings of this study run counter to the fundamentals of the theory, as the risk-reward-relation and size effect are both violated. However, common theoretical understanding is partly respected, higher quality of the stock relates to lower volatility in stock returns, which is observed in the empirical analysis with superior portfolio performance of stocks with high-quality and low volatility.

The base of the study is in findings and methodology of Ang et al. (2009) as they report that the high idiosyncratic volatility delivers low returns (-1.31% per month for long-short portfolio) across 23 developed markets for the sample period 1980-2003. Ang et al. were the first to examine low-volatility anomaly at the individual stock level in cross-sections besides portfolio-level analysis by aggregating stocks based on the volatility of each stock and then measuring portfolio performance and characteristics. Most recent papers provide evidence that the anomaly is found across the Global stock market and is sensitive to an applied weighting scheme. The contradiction remains in the United States as my findings for the later sample period of 2001-2017 do not follow earlier reported results (Ang et al., 2006) for the period 1980-2003.

To motivate the topic further, as the market macro and microstructure are under constant development, past volatility's predictive power for the future returns remains interesting. Lately, the accelerated digitalisation and need for automation have created the highest demand for the change. Currently, the majority of the trades are executed by machines with dramatically increased trading volumes. Algorithms generate 20-30% of all trading in the leading stock exchanges, which is even increased by the human-originated but machine-executed trading activity. Advanced trading technologies and methods should increase traders' capability to capture observed abnormal returns from the market. Nevertheless, past research is mostly based on longer sample periods covering multiple decades, and therefore, those datasets just possess limited exposure to the current algorithmic era. I include the recent past of history by choosing the sample period from the beginning of 2001 to the end of 2017. Such a sample period is also supported by Finnish history with the euro as currency since 1999. Nordic leading stock exchanges are included in the main sample and in addition, the United States stock universe is used as a reference. The primary sample period is divided into three subsamples: before, during, and after the financial crisis.

Researchers have tried to explain the anomaly with multiple economic and technical mechanisms but have failed to find widely agreed explanation. None of the proposed explanations thoroughly explain the low-volatility anomaly. On multiple occasions, the literature finds a relationship between size and low-volatility, specifically on an idiosyncratic level, which then emerges as the primary motivation for the study. The aggregated Nordic market is relatively limited in terms of size and liquidity. The analysis primarily concentrates on IVOL, and its relation to size, and is not liquidity research. However, it is natural to mention liquidity in the context as literature often connects small stocks to less trading activity. To overcome the role of liquidity without examining it further, I take it as a given that small stocks could dominate, explaining multiple inefficiencies as those stocks represent a smaller portion of total trading activity and market participants leave some parts of their possible investing universe with less attention, at least for some periods. Among large stocks, inefficiencies tend to diminish, possibly due to higher trading activity. Therefore, I provide a comprehensive analysis of the Nordic stock universe as a whole but also to large and small stocks separately. In the last section of the analysis, I introduce the twist of the paper by controlling IVOL effect with quality characteristics. Asness et al. (2014, 2018) provide a comprehensive quality measure, which I apply only to examine large cap, the largest quintile of the Nordic stock universe. By doing it with equal-weighted returns, I assume that among the largest stocks, an investor has roughly equal liquidity and depth of the order book available and therefore, size should not play a role. After examining equal-weighted returns for the large cap, one could justifiably question whether the IVOL survives control for quality with value-weighted portfolio returns among large cap stocks. That is indeed a good question and remains available for another thesis.

For further motivation, as sophisticated investors are benchmarked against broad market performance, and therefore unable to fully exploit all observed arbitrage opportunities (Chan et al 2002; Baker et al, 2011), it is reasonable to assume that some of the future returns are still available for systematic harvesting, via IVOL or another strategy.

2 Literature review

Regarding previous empirical findings, I primarily concentrate on the idiosyncratic volatility anomaly and cover the literature in which authors have attempted to address the anomaly in a theoretical or empirical context. Framework for the thesis builds up from the classical finance theory as risk-reward relations and portfolio theory are considered as a base and are either used, further discussed or questioned in the empirical findings and detailed analysis. As John Cochrane of Chicago once described, "Literature reviews have gotten way out of hand," I follow this advice by keeping the backgrounds short and by assuming the reader is familiar with the central development of the economic theory.

Background for the theories applied in the thesis comes from the capital asset pricing model (CAPM) from Markowitz (1959), Sharpe (1964), Lintner (1965), and Fama and French (1992, 1993) in which higher systematic risk should be compensated for and rewarded with a higher expected return. The theory suggests that idiosyncratic risk should be left for diversification and therefore should not be priced. After Black et al. (1972) and Fama and Macbeth (1973), among others, provided empirical evidence that the security market line is flatter than CAPM suggests, the control for size and value was introduced by Fama and French (1992) with their Three-Factor Model (FF-3). In this study, I primarily use a 3-FF model while other multifactor models are left for speculation purposes.

2.1 Idiosyncratic volatility and low-volatility anomaly

Stock return variation is referred to as total volatility, which is defined by two, systematic, and idiosyncratic components. Idiosyncratic volatility (IVOL) refers to the idiosyncratic component of the variation, explaining a stock-specific, and not marketwide, part of the variation. According to CAPM (Lintner, 1965), arbitrage pricing theory APT (Ross, 1976) or later introduced multifactor models (Fama and French, 1992, Carhart, 1997), idiosyncratic volatility should play no role in stock returns. Arguments from Merton (1987) and Malkiel and Xu (2002) are aligned with the classical understanding of risk and reward as they respect economic intuition and suggest that idiosyncratic volatility should have a positive effect on stock return, if anything.

However, Ang et al. (2006, 2009) provide significant contradictory empirical evidence from the United States and the rest of the G7 countries, while in other 23 developed markets, the effect remains visible with less significant numbers. They find that stocks with past low idiosyncratic volatility deliver superior returns compared to high volatility stocks. The phenomenon is called low-volatility anomaly, and while it deviates from the economic intuition of getting rewarded from the risk, it also violates central asset pricing theories of financial literacy. Ang et al. (2009) examine the cross-sectional relationship between expected returns and the IVOL effect extensively; they control for FF-3 loadings, size and book-to-market, Jegadeesh, and Titman's (1993) momentum. For the sake of robustness, they account for leverage (Johnson, 2004), liquidity, analyst forecast dispersion, bear and bull markets, stable and volatile periods, and multiple IVOL estimation periods. Based on one of their conclusions, the international negative spread between high- and low-volatility stocks co-moves with the IVOL spread observed in the U.S. stock market.

Ang et al. (2009) use realised idiosyncratic volatility from the past as their IVOL estimate and compare multiple estimation periods from daily and monthly returns. The effect is strongest with the shortest one- and three-month estimation periods estimated from daily returns. Significance decreases monotonically towards 12- and 36-month estimation periods. Other IVOL estimation methods are introduced, for example, in the paper from Fu (2009) in which they use monthly returns and an exponential GARCH[3] model for the estimation to account for the time-varying nature of idiosyncratic volatility. IVOL portfolios from Fu's EGARCH model leads to contradictory results by showing positive spread for a high-low portfolio with diminished and insignificant alphas. In general, past realised volatility is known to be one of the best performing measures to be used as a proxy for future volatility. If compared to past realised volatility, EGARCH- and GARCH-models lack the predictive performance and perform similarly to estimates from a simple autoregressive model (Stambaugh et al., 2015; Jin, 2013). Furthermore, according to evidence provided by Fink et al. (2012), significant findings in Fu (2009) diminishes if the forecast model is determined correctly and misspecification regarding forward-looking return observations is excluded from the forecast model. Baker et al. (2011) introduce a requirement for benchmarking as an explanation for IVOL. Sophisticated money manager performance is measured against known broad market

[3] GARCH refers to *generalised autoregressive conditional heteroskedasticity*, a statistical time-series model assumes that current error term is a function of actual error terms from previous periods. As the true conditional variance of time series is unobservable, Fu (2009) propose that their estimation method ensures better estimation of the data generation process property.

benchmarks, and therefore, these institutional investors are equipped with limited ability-exploiting observed excess returns on IVOL. Chan et al. (2002) provide the base for Baker et al., as they report that mutual funds tend to shift styles if a fund's performance is lacking against a broad market benchmark, which indicates that a broad market benchmark is a judge which justifies the existence of the fund or vice versa.

One of the recent and most prominent research attempts for IVOL comes from Stambaugh et al. (2015), in which the IVOL effect is explained by mispricing of the stocks. The story is supported by earlier findings (Chan, 2002; Baker et al., 2011) on limitations from benchmarking as they propose that explanatory power comes from arbitrage asymmetries, indicating that the investors are left with relatively limited ability to take short positions compared to long positions. They construct a mispricing variable to estimate the arbitrage asymmetry effect and report that after value-weighting, accounting for size, they are clearly left with less than the previously reported abnormal return of -0.26% (t-stat 1.88) per month for a long-short IVOL portfolio. Negative IVOL spread is observed among overpriced stocks, but spread becomes positive, and therefore anomaly becomes non-existent among underpriced stocks. However, their findings on arbitrage asymmetry leave space for speculation in terms of idiosyncracy, relationship to size, and possible omitted variable bias. As the analysis in Stambaugh et al. (2015) is based on portfolio returns of a double sort on mispricing and IVOL, one could argue that especially the idiosyncratic effect is therefore not measured. Actually, they even show self-criticism as they report that IVOL examination against under or overpricing in cross-sectional regression is not possible due to the fact that their mispricing-function is not known a priori and therefore recognition of underpricing or overpricing is not available. By dividing the stock universe to sorted portfolios, they actually fail to extend their analysis to idiosyncratic level.

2.2 Role of size and weighting scheme

The role of the size in asset pricing theory has been under serious consideration recently as the size factor has lost its explanatory power after the introduction of the five-factor model. In 1993, Fama and French originally included size-factor (SMB) in their three-factor model with the other two factors, market risk (MKT) and value factor (HML). In 2015, the five-factor model was introduced by Fama and French, adding profitability (RMW) and investments (CMA) in the 3-FF model. Some evidence is available that the newly introduced profitability-factor in 5-FF captures part of the variation which was earlier explained by the size-factor. It is fair to make a statement that the Fama French three-factor model was recognised widely, and it deserved the role as the reference model. However, four-factor (Carhart, 1997), five-factor (Fama and French, 2015), or the latest six-factor models (Fama and French, 2018) are recognised and discussed without such academic and industry-wide celebration that we have seen with 3-FF. After Banz (1981) published his work on size effect, the premium basically disappeared in the U.S. stock market. Bali and Cakici (2008) argue that small firms primarily drive the IVOL effect and therefore, the robustness of the Ang et al. (2006) is questioned. Bali and Cakici argue further that the IVOL is correlated positively with size and Amihud illiquidity. Huang et al. (2010) and Han and Lesmond (2011) explain the Ang et al. (2006) IVOL effect with return reversals and liquidity. Further, Alquist et al. (2018) argues that the size premium may not have even existed in U.S. stocks. Instead of size, they offer explanations like profitability and low volatility, and besides that, the size effect is dominated by a January effect.

Despite that Ang et al. (2006) are the first to measure IVOL effect in individual stock level, they also examine portfolios with value-weighted returns. Bali and Cakici (2008) confirm the findings with value-weighted portfolios, but with equal-weighted portfolios, the effect is not observed. Fama and French (2008) suggest that equal-weighting prevents a few large stocks driving the results. While explaining the size effect, controlling for quality, Asness et al. (2018) together with Miller (1977) develop the size story by stating that constraints for example on short selling among small stocks implies that the prices mostly reflects only the opinions of optimists. Required liquidity for short leg of IVOL implementation would dry in that case, bring the prices down, and cut the observed returns of the long-short portfolio. Therefore, understanding the role of small stocks is crucial for IVOL success.

2.3 Role of quality

According to Asness et al. (2014), high-quality stocks are profitable, growing, and safer than junk stocks, and quality appears to hedge of market distress as it performs well during down markets. As the size (SMB) has just modest, but significant, role in their U.S. and global samples, they show that controlling for quality[4] effect (QMJ), the size becomes large and highly significant. Furthermore, they intuitively bind quality with size and volatility by providing

[4] Quality-minus-junk (QMJ) factor was developed by Asness, Frazzini, and Pedersen (2014), described as portfolio that invests long quality stocks and shorts junk stocks which produces high risk-adjusted returns. Price of quality is time-varying and benefits from flight-to-quality during crises.

evidence that small stocks are junky while big stocks represent quality, and junky small stocks are more volatile yet being profitable. Also, they claim that QMJ is a useful factor in right-hand-side applications to test if another phenomenon is driven by quality. To motivate the quality-IVOL-story a bit further, while Asness et al. (2014) succeed to convince quality's role as a robust RHS-variable, they fail to demonstrate that prices vary cross-sectionally enough with quality to propose that QMJ would deserve a role as a risk factor.

Asness et al. (2018) strengthen their earlier findings by showing that without control for quality, the variation left unexplained by weak size effect is explained by other correlated factors. They propose that quality should be controlled to understand size effect thoroughly: the economic significance of the size becomes on par with value and momentum. The statement is supported by the robust findings of 30 different industries and 24 international equity markets.

2.4 Other possible explanations for IVOL

Other recent explanations for low-volatility anomaly include items such as lottery-trading related effects (skewness, coskewness, expected idiosyncratic skewness, maximum daily return, and retail trading proportion) or effects originated from market frictions (one-month return reversal, the Amihud illiquidity measure (Amihud, 2002), zero-return proportion, and bid-ask spread) which alone or together explain the abnormal returns only to some extent (Hou and Loh, 2016).

According to IVOL out-of-sample examination from Spiegel and Wang (2005), both liquidity and idiosyncratic risk determine stock returns, the effect from conditional idiosyncratic risk dominates clearly the effect of liquidity. Birru (2018) offers another view explaining long-short anomaly returns with Monday-Friday effect. Returns in anomalies with speculative short leg experience highest returns on Monday and vice versa for long leg while the opposite pattern is observed on Friday. They suggest that the psychology literature is consistent with the effect by recognizing mood increase on Friday and decrease on Monday. In addition to behavioural aspects of human market participants, and to motivate further the sample period covering a recent couple of decades in the Nordic market, Chordia et al. (2014) provide evidence that improvements in trading technology and the cost of transacting have led to increased liquidity and explosion in trading volumes. It is argued that increased liquidity and trading activity lead to attenuation of earlier observed equity anomalies due to increased arbitrage.

3 Development of Hypotheses

To answer a broad question of the thesis topic regarding to Nordic idiosyncratic volatility anomaly existence, I establish three hypotheses and propose a couple of additional questions. Hypotheses should be understood as a base and framework for the analysis, but they are not directly referred in the work later. Therefore, the main findings are already provided for each step within the hypothesis development as I am willing to ensure that the logic is fully observable.

First, I confirm that idiosyncratic volatility and expected returns do not respect known financial theory in Nordic markets and therefore low-volatility stocks outperform high-volatility stocks. To deepen the level of detail, the analysis concentrates on the individual stock level, therefore idiosyncratic volatility is mainly under examination while total volatility with its systematic characteristics deserves less attention. According to prior research (Ang et al, 2009), the IVOL effect is observed in multiple international markets for an earlier sample period. I test for the following hypothesis:

> *H1: Low-volatility anomaly using Fama French three-factor risk adjusted returns exists in Nordic aggregated market and its constituents.*

As provided within the results, IVOL has significant explanatory power in the aggregated Nordic cross-section, it has negative relation to future expected returns which then confirms the existence of the anomaly. IVOL has the most explanatory power in Sweden, it is significant in Finland and Denmark, while Norway is contrast with insignificant IVOL. Contrary to the findings of Ang et al. (2009), a U.S. reference sample does not show significant IVOL.

Second, I test the IVOL against variation related to size. Both sizes have the ability to explain the anomaly directly, but also the role of the size effect itself has motivated earlier research attempts. Lack of liquidity, less diversification, or shorting constraints are all claimed to relate to small firms and therefore to the size. According to prior research, on average IVOL has lost some, but not all, of its explanatory power after control for size is introduced. I test the following hypothesis to answer whether the size effect is actually the dominant risk factor and explains the idiosyncratic volatility anomaly in the Nordic region:

> *H2: IVOL is significant with equal and value-weighted returns and survives control for size.*

Results from equal and value-weighted regressions typically deviate from each other in a global context. For the Nordic market, I report that the IVOL coefficient strengthened in magnitude with slightly less explanatory power, still remaining significant. All Nordic constituents, including Norway, are significant, U.S. reference samples remain insignificant.

Third, as the size fails to explain all of the variation related to IVOL, I next concentrate on quality. Asness et al (2018) reports that after controlling for quality (or junk), the size effect is shown to gain its robustness. Quality, especially QMJ, has a twofold connection to IVOL. Either controlling for quality, the size is shown to gain power and therefore size's explanatory power on IVOL should also further increase, or either, by following basic intuition, that investor seeks and picks out the stock with better quality and less volatile expected returns. Asness et al (2014) finds that smalls are junky relative to big firms, small firms are more volatile and yet to be profitable. Following hypothesis is tested to answer whether the idiosyncratic volatility anomaly survives the control for quality:

H3: IVOL remains significant after adding the QMJ factor in the model.

I examine Nordic large cap stocks for quality and report that IVOL succeeds at surviving with a coefficient significant in conventional level.

Finally, I continue by proposing the following questions to extend analysis further towards implementation requirements of the strategy:

Where is the effect located in the Nordic market? Among which kind of stocks? Is it investable? How is the performance?

To answer the final questions, I examine multiple time periods, apply portfolios with double sorts, first on IVOL and then for size or quality, provide risk exposures relative to regional 3-FF. As testing the hypotheses could be seen as a logical process to build foundation for strategy implementation, then, by answering to these additional questions I gain preliminary information if the anomaly would have been available for investing in a real-life context. IVOL survival among Nordic large cap stocks is an interesting starting point for portfolio construction and performance analysis. However, further sorting by country or industries among Nordic large cap, including challenges with limited numbers of stocks, is left for another research attempt. Industry-specific analysis would be motivated by the fact that, during the techno bubble Nordic market saw a period of major single industry dominance when Nokia and Ericsson together represented over half of total Nordic market capitalisation.

4 Data and Methodology

4.1 Data

Market and fundamental data for the study is collected from Refinitiv Datastream and consists of stocks listed in NASDAQ OMX Copenhagen, NASDAQ OMX Helsinki, NASDAQ OMX Stockholm, and Oslo Börs. Other than the main Nordic stock exchanges are excluded. The primary sample is from January 2001 to December 2017. Data is fetched from the beginning of the year 1999, the first couple of years from 1999 until the beginning of 2001 are used for variable estimation or formation. Rationale behind the sample period is as follows: Finland changed over to the euro as official book currency in January 1999, even though both the former Finnish markka and the euro had legal tender status until February 2002. Also, extreme market conditions from the techno bubble were converging back to normal, if we can say so, right ahead of main sample start in January 2001. The sample is constructed with euro-denominated returns to serve the purpose of investors investing in euros. The sample is constructed for each step of the analysis so that all relevant variables are required, and no missing values are interpolated or extrapolated. Data for the U.S. reference sample comes from Datastream as well, equal variable names are used to fetch the data, to preserve consistency with Nordic analysis. Further, extension to analysis are provided as I include the U.S. dataset of 1980-2003 covering the similar period with Ang et al. (2009) as a comparison.

Datastream common and primary flags are required for the stock from equity universe to be included in the sample. Possible survivorship bias is avoided by including all, also currently dead, stocks in the sample. Stocks with currency in the market or fundamental data other than one of the expected for each country were filtered out. A return index (RI) is used for return calculation, the variable represents total return of the stock including dividends and accounts for adjustments. Returns, used for left-hand-side variables, are simple returns calculated as

$$r_{i,t} = \frac{RI_{i,t} - RI_{i,t-1}}{RI_{i,t-1}} \tag{1}$$

After calculating returns, outliers are filtered out by replacing return observations above 300%, or below -100%, with N/A. A minimum market equity value of EUR (USD) 5 million is required for the stock and for the period to be included in the sample. These direct filters clean the most extreme outliers from the sample while during the analysis, indirect filtering continues cleaning as the same requirements are applied for lagged values such as the past six months return, lagged book equity, or lagged book-to-market. Detailed variable definitions are included in the Appendix.

For the Nordic aggregated market and for each Nordic country, I use euro-denominated values. Local currencies are expressed in euros by using a mid-spot rate for the last day of the period. All analysis regarding the U.S. is based on values expressed in U.S. dollars. By doing so, I ensure and audit the model and methodology implementation used for the analysis. Firstly, U.S. findings are directly comparable with the findings from previous research. Secondly, as the analysis of the thesis is based on an otherwise equal procedure for Nordic and U.S. data, I can bind my work with prior research without being dependent on used currency. In addition to a base case with euro-denominated values, I do some preliminary analysis with values expressed in local currencies individually for Finland, Denmark, Norway, and Sweden. However, Nordic markets seem to be integrated at such a level that hardly no deviation is observed between local currency and euro-denominated results. Further analysis related to local currencies is not included in the thesis.

Delisting returns are represented as they occur in Datastream data. Datastream data, even after filtering I have applied, is pure market data with all the quirks and flaws while prior literature mostly refers to the data from CRSP[5]. The dataset from CRSP is meant for education and research use with multiple kinds of entry requirements and cleaning methods in place. While the pure market data requires cleaning, the CRSP may suffer from a fairly-explained approach built into the construction of the dataset. Possible shortcomings are delisting returns (Shumway, 1992) or the bias from the fact that new stock is required to trade for plenty of time before it's included in the dataset. Relating to the long-short portfolio approach, extreme portfolios may deserve relatively too much attention with too nasty, and therefore not-representative, content. Therefore, middle portfolio performance should be analysed and aligned as well as both long and short-leg performance individually. A risk-free rate of return is a European risk-free interest rate fetched from the Kenneth French database[6].

[5] Center for Research in Security Prices (CRSP) at the University of Chicago

[6] Kenneth French's data base provides risk-free rates among market and Fama French factor portfolios and related returns, available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 1. Descriptive statistics, sample
Main sample is filtered and described below. EUR-nominated data is used for Nordic. USD-nominated data from the United States is used for benchmarking and processed equally with the Nordic data. *Number of firms* is the average number of valid firms included in the sample for each moment. *With full coverage* is the number of constituents which are present in all moments in the sample. *Size* describes the statistics of time series market equity. Main sample is divided in three subsamples for further analysis.

	Number of firms	With full coverage	Size (m)			
			Mean	25^{th}	50th	75^{th}
Panel A. Main sample 2001/01 - 2017/12 (204 months)						
Nordic	754	303	1062	28	94	441
Finland	122	65	1271	42	150	667
Denmark	151	75	1092	26	79	491
Norway	168	52	934	36	111	446
Sweden	311	110	982	24	79	382
United States*	3763	1522	4292	131	536	2104

* *U.S. dollars*

Table 1 reports descriptive statistics of the samples used for the analysis. While aggregated Nordic market is one fifth of U.S. in terms of number of firms, Sweden has the most firms in Nordic with lowest median size of EUR 79 million compared to Nordic median if EUR 94 million. Mean size for each country is around EUR 1 billion. Even though U.S. is expressed in dollars, we can observe that median (mean) size for U.S firms is roughly five (four) times of Nordic correspondents. Information for each subsample is provided in the appendix.

As seen in Table 2, volatilities increase systematically if the estimation window is extended from short one-month to any of longer periods. Idiosyncratic volatilities estimated from daily returns, seen in Table 2 Panel A, are in similar level in Nordic and in U.S. while Finland shows the lowest and Norway the highest volatilities in the Nordic region. With the longer estimation periods of 12 to 36 months estimated from monthly returns in Panel B, U.S. volatilities are higher than volatilities observed in the Nordics. Sweden dominates Nordic aggregated volatility as Swedish firms, with equally weighted exposure, form the majority of constituents for time-series mean calculations. Furthermore, U.S. monthly estimated idiosyncratic volatilities are on average 11.3% above the Nordic level. Subsample volatilities are included in the appendix.

Figure 1 illustrates volatility development over time for each market, showing similar patterns especially around the financial crisis. We can see that the overall magnitude of volatility, after sharp increases and decrease in the end of 2011, is attenuated to the stable level in all markets.

Table 2. Volatilities, multiple estimation periods

Main sample consist of Nordic EUR-nominated data. USD-nominated data from the United States is used for benchmarking and processed equally with the Nordic data. Table describes total volatility (TVOL) and idiosyncratic volatility (IVOL) estimated from daily and monthly returns. All volatility values are annualised by multiplying estimated percentage of volatility with square of 250 for daily volatility and with square of 12 for monthly volatility. 1m to 36m describes prior window used for volatility computation. *TVOL* is a time-series mean of simple standard deviation measured from excess return of each stock. *IVOL* is defined as time-series mean of simple standard deviation measured from variation in error-terms of FF3-factor estimation for each stock.

	Daily returns				
	TVOL (%)		IVOL (%)		
	1m	12m	1m	6m	12m
Panel A. Main sample 2001/01 - 2017/12 (204 months)					
Nordic	41.18	44.66	35.05	39.11	39.31
Finland	34.15	37.17	28.56	32.45	33.12
Denmark	34.19	37.62	29.61	33.81	34.25
Norway	45.81	50.17	38.62	43.79	44.23
Sweden	45.17	48.51	37.14	40.83	40.81
United States	43.19	47.07	35.65	40.28	41.08

	Monthly returns				
	TVOL (%)		IVOL (%)		
	12m	24m	12m	24m	36m
Panel B. Main sample 2001/01 - 2017/12 (204 months)					
Nordic	38.64	40.28	28.24	31.19	31.91
Finland	31.68	33.27	22.79	25.50	26.41
Denmark	32.38	33.75	24.61	27.53	28.50
Norway	42.77	44.52	31.09	34.20	34.84
Sweden	42.63	44.57	29.67	32.80	33.60
United States	41.95	43.89	31.25	34.92	36.20

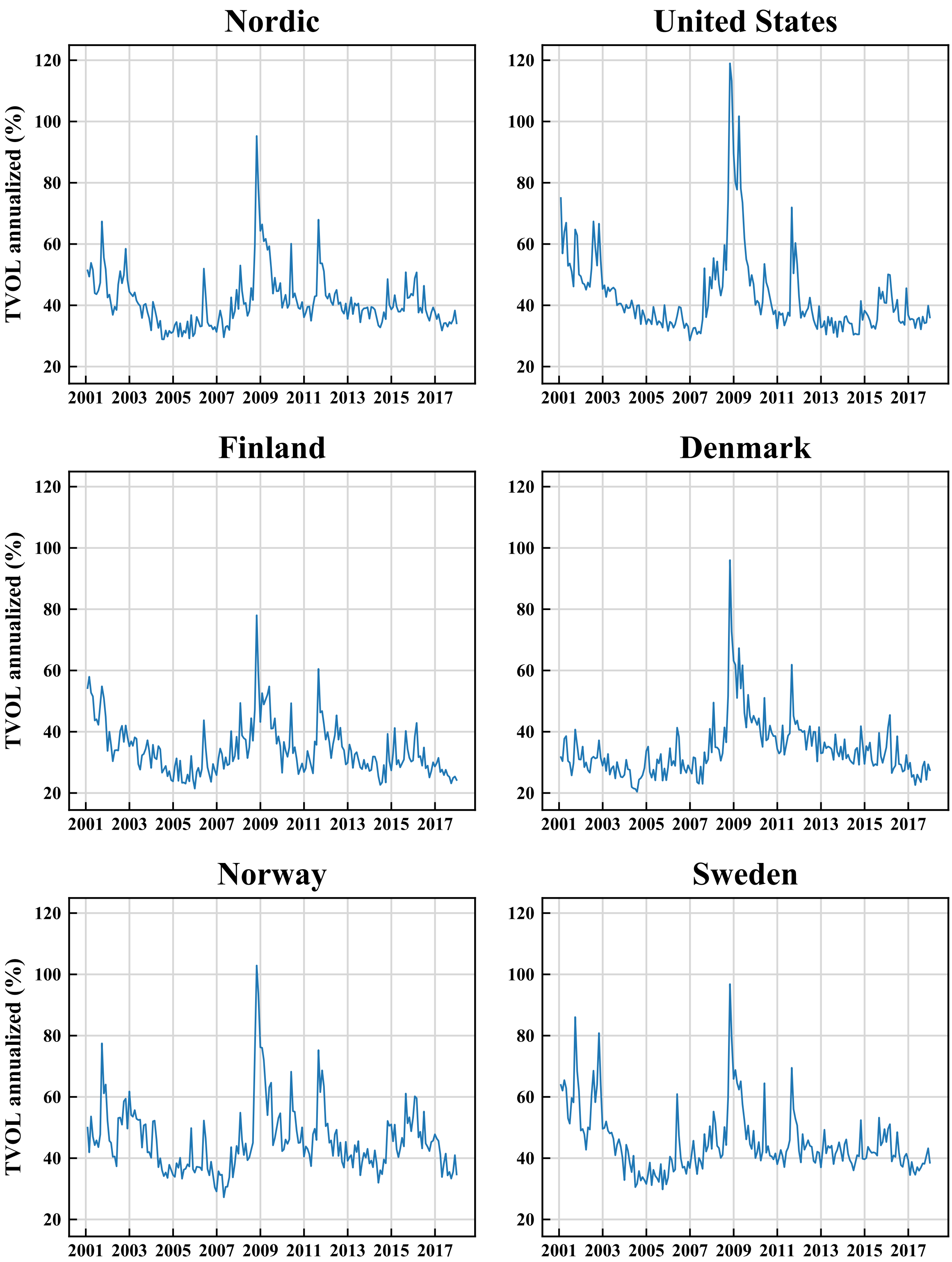

Figure 1. Total volatility, annualised
Figure illustrates annualised total volatility (*TVOL %*) over sample period 2001/01-2017/12. TVOL is measured as standard deviation in excess daily returns over prior one month. The time series is constructed by averaging cross-sectional total volatilities of the constituents with market equity value above EUR (USD) 5 million. Realized volatility is annualised with multiplying daily volatility by 250 squared. Nordic returns are denominated in EUR currency, U.S. returns in dollars.

Massive quantitative easing and large-scale asset purchases by central banks could be seen as possible explanations for a less volatile environment. At least the timeframe fits into the *new normal*[7], an era of extremely low interest rates and unconventional monetary policy.

4.1.1 FF-3 factor construction

I calculate regional Fama-French three-factors for the Nordic market. Previous research has used pan-European factors which could lead to bias as contribution from the Nordic market into European level is limited. I follow procedure described in Fama and French (1993) and compute FF-3 risk factor returns MKT, SMB, and HML for aggregated Nordic market and each country individually. By using factor returns especially calculated for the specific market, and not using ready-made factor returns, I seek to fine down the analysis to capture all relevant variation. For Nordic market equity, the value median is used as a size breakpoint for the SMB calculation, for HML, 30^{th} and 70^{th} percentiles are used as breakpoints. For the U.S. sample, equal breakpoints computed from NYSE stocks are applied to the whole dataset. Each factor return is calculated monthly, using value-weighted portfolio returns, and then averaging the portfolio returns following the procedure described in the original paper. In the appendix, I provide FF-3 factor calculation benchmarks by plotting U.S. factor returns fetched from Kenneth French, from AQR, and from my proprietary calculation. These factor returns are plotted and available in the Appendix. Table 3 reports FF-3 factor monthly returns used in the analysis.

Table 3. Fama French Three-factor returns
Table reports FF-3 risk factor excess returns (%) in monthly terms computed following Fama and French (1993) for each market. SMB refers to small-minus-big and HML to high-minus-low factors. MKT is market portfolio for the market, including equity universe less REITs. Sample period for all markets is from January 2001 to December 2017. Nordic and its constituents are expressed in EUR, U.S. in USD.

	Nordic	Denmark	Finland	Norway	Sweden	United States
Panel A: FF-3 factor excess returns						
FF-3 MKT	0.553	0.846	0.297	0.798	0.604	0.545
FF-3 SMB	-0.032	-0.394	0.116	-0.351	0.026	0.302
FF-3 HML	0.526	0.134	0.826	0.328	0.792	0.278

[7] "The Fed Is Irrelevant: Low Interest Rates Are The New Normal", Forbes. February 1st 2019, available at https://www.forbes.com/sites/greatspeculations/2019/02/01/the-fed-is-irrelevant-low-interest-rates-are-the-new-normal/#46ee483176ae, loaded 12.2.2019

4.1.2 Quality factor construction

I construct the quality-minus-junk (QMJ) factor by following the methodology described in Asness et al. (2014). QMJ includes three composite quality measures: *profitability*, *growth*, and *safety*. Individual components are averaged to compute a single overall quality score for each firm. Each of the three quality components include five to six individual measures which are converted to ranks and standardised to z-scores[8] on a monthly basis.

Profitability is constructed from six individual measures, gross profits over assets (GPOA), return on equity (ROE), return on assets (ROA), cash flow over assets (CFOA), gross margin (GMAR), and cash earnings (ACC) by averaging:

$$Profitability = z(z_{GPOA} + z_{ROE} + z_{ROA} + z_{CFOA} + z_{GMAR} + z_{ACC}) \tag{2}$$

Similarly, the *growth* is measured as the five-year (Δ) growth in each of five profitability measure, excluding accruals. I first compute growth over last five years, and in case of lacking data, I then try with growth over subsequent four- and three-year periods:

$$Growth = z(z_{\Delta GPOA} + z_{\Delta ROE} + z_{\Delta ROA} + z_{\Delta CFOA} + z_{\Delta GMAR}) \tag{3}$$

Safety is composed from five individual measures, indicating that safe securities are the ones with low beta (BAB), low leverage (LEV), low bankruptcy risk (O-score, Z-score), and low volatility on ROE (EVOL):

$$Safety = z(z_{BAB} + z_{LEV} + z_{O} + z_{Z} + z_{EVOL}) \tag{4}$$

in which BAB is the "Betting Against Beta" factor from Frazzini and Pedersen (2014), computed specifically, like all other measures mentioned above, for the Nordic market individually. Detailed approach is available upon request and is not include in the thesis.

---

[8] $z_x = (r - \mu_r)/\sigma_r$, where μ_r and σ_r are cross sectional mean and standard deviation

Finally, single score is constructed by combining three subcomponents into a single quality score for each firm for the period:

$$Quality = z(Profitability + Growth + Safety) \tag{5}$$

QMJ factor returns are reported in Table 4. Construction of the factor is data demanding and basically drops the majority of smaller stocks from the sample. The Nordic-aggregated sample includes on average 500 stocks while Danish and Finnish markets constitute only 84 stocks. To compare, Finnish markets' largest quintile is on average approximately 130 stocks. Direct benchmark for the period is not available, the closest is AQR Betting Against Beta: Original Paper Data[9] with samples ending in 2012/03. On average, their USD-denominated mean monthly returns for equities for the period 2001/01-2012/03 are Finland 2.21%, Denmark 0.97%, Norway 0.96%, and Sweden 1.24%, making the average for the Nordic region 1.34%. In my sample, returns are robust and significant with smaller coefficients.

Table 4. QMJ quality-minus-junk factor returns

Table reports QMJ factor excess returns (%) in monthly terms computed following Asness et al (2014) for each market. *QMJ* refers to quality-minus-junk factor which is long in quality and short in junk. Factor includes equity universe less REITs. *N* is average number of stocks in sample. Largest stocks of each market are included as QMJ requires multiple variables over multiple timespans for construction. Sample period for all markets is from January 2001 to December 2017. Nordic aggregated market and its constituents are expressed in EUR. Robust Newey-West t-statistics with four lags are reported in square brackets.

	Nordic	Denmark	Finland	Norway	Sweden
Panel A: QMJ factor excess returns					
QMJ	0.721**	1.373***	0.227	1.186***	0.827**
	[2.36]	*[3.29]*	*[0.54]*	*[2.85]*	*[2.08]*
N	500	84	84	115	217

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

[9] AQR publishes datasets for the papers from authors contributing to academia but also AQR. Betting Against Beta: Original Paper Data is available at AQR website at the time of writing the thesis: https://www.aqr.com/Insights/Datasets/Betting-Against-Beta-Original-Paper-Data

4.2 Methodology

Methodology is twofold. Individual stock level is examined with cross-sectional analysis and after that, holding period returns are estimated analysing single- and double-sort portfolios. Firstly, I look at individual stock level with the Fama Macbeth (1973) cross-sectional regression approach (FMB). The method enables multiple effects to be controlled as either returns, factor loadings, while other related variables are valid for right-hand-side use. By applying FMB, explanatory power of the effect could be estimated at the individual stock level and compared against other effects, the feature of which makes FMB superior to conventional time series analysis. The method is described in detail in the following subsection. Ang et al. (2009) used similar methodology examining international the IVOL effect.

Secondly, to examine the performance of IVOL and assess if the effect is investable in the Nordics, I first construct quintile portfolios by sorting the stocks based on a previous period of IVOL. After the single sort, portfolios are constructed with to double sorts, first on IVOL and size, and then IVOL and quality. Double-sorted portfolios examine IVOL performance among each size and quality segments individually. Portfolio returns are examined with Fama and French (1993) time-series regressions, reporting excess returns, alphas, and risk factor loadings with Newey-West robust t-statistics. I accept that some level of detail is lost after the information is aggregated in 5-by-3 and 3-by-3 portfolios, therefore the effect is first tested in the FMB setting and then provides portfolio analysis as an extension towards implementation requirements of the strategy. Ang et al. applied the portfolio approach in their first paper in 2006, however, after their work was published and questioned, they extended their analysis to the individual stock level by applying the FMB regression analysis for the 2009 paper.

The methodology used in the analysis, by way of its construction, should prevent data snooping in the first place. The base for applied methods is either in robust economic theory, or they are otherwise tested with another dataset in prior literature. Use of the methods for the Nordic dataset could be seen more like another out-of-sample test. Also, reported results include full analysis, for example in monotonicity, providing details for the reader’s personal judgement.

4.2.1 Volatility estimation and multiple estimation periods

As the present volatility is unobservable, I follow Ang et al. (2006, 2009) and estimate the volatility from past one-month daily returns. In addition, to examine the sensitivity of the volatility estimation period, estimation periods of the past 6 and 12 months using daily returns and past 12, 24, and 36 months using monthly returns (Bali and Cakici 2008) are measured. I require a minimum data density of 50% for the rolling estimation window.

By following Ang et al. (2009), I use both, total and idiosyncratic volatility to reveal the source of realised volatility risk premiums. Total volatility estimation for multiple estimation periods is sample standard deviation described as

$$TVOL_{i,t+1} = \sqrt{\frac{1}{N-1}\sum_{k=0}^{N-1}\left(r_{i,t-k} - \bar{r}\right)^2} \qquad (6)$$

in which *N* describes the number of observations in the estimation periods, $r_{i,t}$ is a simple raw return for the stock in time *t*. The timing is worth mentioning, for the analysis, TVOL in time *t* is the TVOL estimated for the previous moment, using daily (monthly) raw excess returns from the prior period.

For idiosyncratic volatility estimation, first, return time-series for each stock is regressed on FF-3, the regression takes the form

$$r_{i,t} = c + dMKT_t + eSMB_t + fHML_t + \varepsilon_{i,t} \qquad (7)$$

where $r_{i,t}$ denotes simple excess return for stock *i* at time *t*, *MKT*, *SMB*, and *HML* are factor returns constructed for each particular market following FF-3 procedure (Fama French, 1993). Idiosyncratic volatility is a sample standard deviation estimated from the error term of FF-3 model and is expressed as

$$IVOL_{i,t+1}^{FF-3} = \sqrt{\frac{1}{N-1}\sum_{k=0}^{N-1}\left(\varepsilon_{i,t-k} - \bar{\varepsilon}\right)^2} \qquad (8)$$

in which $\varepsilon_{i,t}$ is the error left unexplained after the FF-3 risk factors, the timing of which is similar to TVOL described above.

4.2.2 Fama Macbeth cross-section

FMB cross-sectional regression is computed in three stages in the setting of the study. In the first stage, the time series of past returns are used to estimate IVOL and FF-3 loadings for each stock and for each moment *t*. Multiple IVOL estimation periods and FF-3 definition for the Nordic market are covered in the respective subsections. The second stage is for cross-sectional regression in which each stock excess return is regressed with an individual stock's own idiosyncratic volatility, FF-3 factor loadings, logarithmic size, book-to-market, and lagged return over the past six months. The FMB cross-sectional regressions take the following form:

$$r_i(t,t+1) = c + \gamma IVOL_i(t-1,t) + \lambda'_\beta \beta_i(t,t+1) + \lambda'_z z_i(t) + \varepsilon_i(t+1) \qquad (9)$$

Where $r_i(t,t+1)$ is an excess return expressed in percentage terms for stock *i* from month *t* to *t+1*, $IVOL_i(t-1,t)$ is idiosyncratic volatility estimated over the previous month's daily excess returns from *t-1* to *t*, $\beta_i(t,t+1)$ denotes a vector of risk factor loadings (β MKT, β SMB, β HML) for each stock *i* for the period from *t* to *t+1*, $z_i(t)$ is vector of firm characteristics observable at time *t*. Coefficient γ on idiosyncratic volatility expressed as annualised volatility describes explanatory power of IVOL and should be zero and insignificant with a correctly specified factor model. In the third stage, coefficient time series from the second stage are used to test whether coefficients are significantly different from zero. Newey-West (1987) robust t-statistics with four lags are computed to account for possible serial correlation among coefficient estimates.

Following earlier research (Ang et al, 2009; Black et al, 1972; Fama and French, 1992; Shanken, 1992) contemporaneous factor loadings are used to control for risk exposures. Whereas Ang et al. (2009) use global MKT, SMB, and HML factor returns for risk exposure estimation for international markets, I use regional Nordic and local country specific factors constructed especially for the study. Following Ang et al. (2009) and Daniel and Titman (1997), firm-level characteristics, such as log size, book-to-market, and lagged six months momentum from Jegadeesh and Titman (1993) are included in the regression equation. For the last part of the study, the quality measure QMJ from Asness et al (2018) is added to the right-hand-side of the regression equation. All variables are expressed in local currency for each country, aggregated Nordic analysis is based on euro-denominated values, and values for the United States are measured in U.S. dollars.

With the FMB setting, I first examine IVOL relation with expected returns after accounting for conventional controls. In the later part of the study, the relation of expected returns and IVOL is further challenged by introducing quality. We are especially interested in the front signs and significance of IVOL and QMJ variables. For the size story, the *size* coefficient seen in the regression table is an introduction while the story itself is further developed with a portfolio approach by providing double sort portfolio excess returns, alphas and risk factor loadings.

The original FMB approach assumes equal weight for each observation in a cross-sectional OLS regression setting. To examine the first hypothesis, I compute FMB coefficients with equal-weighted returns. For the second hypothesis regarding size, I use WLS instead of OLS, as a weighted least squares WLS method allows the use of value-weighted settings for return regression. The WLS takes in an observation weighting vector with the requirement that the weights are known a priori. As I compute weights from observable market equity values, the requirement is fulfilled, and method robustness is ensured. The minimisation of the sum of squares with equal weighting (OLS) is expressed as

$$\hat{\beta}_{OLS} = \arg\min_{\beta} \sum_{i=1}^{m} \left| y_i - \sum_{j=1}^{n} X_{ij}\beta_j \right|^2 \tag{10}$$

in which y_i denotes dependent values, X_{ij} contains independent values, and β_j is coefficients from the minimisation. The minimisation of the weighted sum of squares (WLS) becomes

$$\hat{\beta}_{WLS} = \arg\min_{\beta} \sum_{i=1}^{m} w_i \left| y_i - \sum_{j=1}^{n} X_{ij}\beta_j \right|^2 \tag{11}$$

in which $w_i > 0$ denotes the weight of each observation, while the latter part of the equation follows the OLS approach.

4.2.3 Portfolio formation

For portfolio formation, stocks from each region are first sorted in quintile portfolios based on the prior period's IVOL. Stocks with the lowest IVOL go to portfolio number one (P1), while stocks with the highest IVOL go to portfolio number five (P5). Portfolios are reformed and refreshed at the beginning of each month with a holding period of one month. A one-month portfolio refresh frequency follows prior literature and is a relevant average assumption for real-life execution. Euro-currency denominated values are used for all Nordic portfolios. For the U.S. portfolio, I follow an earlier assumption conceived for the thesis and use U.S. dollar-denominated values. Portfolio holding period returns are computed with equal weighting.

After single sort, I do two double sorts, first IVOL and size for each individual country and Nordic aggregated, then IVOL and quality for the Nordic-aggregated market. IVOL is examined among each size and quality segment. Size is divided into three portfolios, *small*, *medium*, and *big* with breakpoints in the 30th and 70th percentiles. Quality is divided in three as well, *junk*, *neutral* and, *quality* with 1/3 and 2/3 as breakpoints. Size sort is based on market equity value in the beginning of the period. For quality sort, I use a QMJ score calculated at the beginning of the period for each stock. For the IVOL-size-sort, a full Nordic sample is divided into 5x3-portfolios, whereas the IVOL-QMJ-sort for the Nordic aggregated market with an average number of 489 stocks is examined with 3x3-portfolio formation.

Regarding the weighting scheme, I choose to do analysis with equal-weighted returns, indicating that each constituent contributes an equal amount in portfolio return calculation and is computed by averaging constituents' returns for the period. Nordic markets include individual large stocks, currently, for example, Novo Nordisk in Denmark, or former telecom giants Nokia from Finland and Ericsson from Sweden. With value weighting, these individual companies would adversely dominate portfolio return so that the analysis would possibly rely only on a few stocks' performance. However, even though preliminary analysis of value-weighted returns provides significant results, more accurate and comprehensive research is beyond the limitations of this thesis work. Value-weighted portfolios are used for constructing the factor returns used in the study, following original authors, market equity value for each stock is used from the beginning of the period and portfolio weights are calculated so that each portfolio weight sums up to one, ensuring that each stock's contribution to the portfolio return follows its relative size among other portfolio constituents. Then, each stock's return is multiplied by the computed weight for the period, finally those return contributions for each portfolio are summed together.

Holding period alphas and factor loadings are calculated following the Fama and French (1993) three-factor model time series regression equation

$$r_i = \alpha_i + \beta_i MKT + s_i SMB + h_i HML + \varepsilon_i \tag{12}$$

in which r_i is a monthly excess return of stock *i* and MKT, SMB, HML are FF-3 factor returns calculated specially for each market.

5 Results

Idiosyncratic low-volatility anomaly exists in Nordic markets. I show that low IVOL stocks outperform high-IVOL stocks after control for size and quality. The relationship of future returns and past idiosyncratic volatility remains significant in the cross-section and provides significant alpha and excess returns for the long-short portfolio which long low-IVOL stocks and short high-IVOL stocks. The effect is especially strong among medium-sized stocks, exists in big, and is not observed among the smallest stocks. In terms of quality, measured as a QMJ score by Asness et al. (2014), *junk* stocks deliver the largest alpha and excess returns, a similar effect is observed among *neutral* stocks while the highest *quality* stocks deviate with insignificant results. Nordic markets show a similar IVOL pattern and the only clear deviation is observed in the Norwegian market with insignificant coefficients but similar front signs in the portfolio analysis for holding period returns. Among Nordic markets, total volatility follows theory with observed risk-return relation while idiosyncratic volatility does the opposite and shows similar relation to future returns than what is reported by Ang et al. (2006, 2009) for the United States from 1980-2003. Subsamples show that *pre-crisis,* the anomaly is only significant in Finland, during the *financial crisis* insignificant in all markets, while the *post-crisis* delivers significant excess return and alpha for the Nordic-aggregated market. Currently for 2001-2017, the United States provides contrary results, IVOL is insignificant in the cross-section, long-short portfolio alpha and excess return are insignificant with inverse front signs. Therefore, to make sure the methodology is implemented correctly, in the appendix I provide extension to the analysis and report significant findings, which are similar to past literature, for the United States for the earlier period of 1980-2003 following Ang et al. (2009). An included comparison also confirms the successful replication of their findings. Hou and Loh (2016) report similar coefficients and significances for their longer sample period of 1963-2012.

I begin the analysis in Section 5.1 with a cross-sectional examination of the IVOL effect in individual stock level regressing future stock returns on lagged idiosyncratic volatility across the Nordic-aggregated market, its constituent countries, and the United States. Section 5.2 provides controls for size, Section 5.3 considers controversial results regarding the U.S., and Section 5.4 reports the IVOL survival with the control for quality. Holding period returns are examined in Section 5.5 for portfolios sorted on previous period IVOL among all stocks and then within each size and quality segments. Robustness of the results and possible shortcomings are discussed in the last subsection, 5.6.

5.1 Low-volatility anomaly exists in Nordics

In Table 3, the Fama Macbeth (1973) regressions equation (4) results confirm that, among Nordic markets, stocks with low idiosyncratic volatility outperform stocks with high idiosyncratic volatility and succeeds to explain future returns with reported significant coefficient -1.250 in Panel A. We are especially interested if the IVOL coefficient is positive or negative, and if so, if the finding significant. Negative coefficients for IVOL confirms that anomaly is observed and indicates that higher positive future excess stock return is explained on average by lower idiosyncratic volatility, and vice versa. Regressions are run with monthly data, on a full sample from January 2001 to December 2017, using prior data, if needed, from 1999 to beginning of 2001 for the formation of independent right-hand-side variables. Nordic regression is run with data including each individual stock, and not by pooling and weighting country-specific results as is seen in multiple earlier research settings. Equal-weighted scheme for returns is used for the regression, following OLS minimisation equation (7). Results are reported using euro-denominated returns for Nordic markets and dollars for the United States. Analysis regarding the U.S. market is left to Section 5.3., as coefficients are not directly comparable due to currency difference.

First, for the Nordic markets, we observe a clear negative relationship between idiosyncratic volatility and average future excess returns with a highly significant coefficient of -1.260 with robust Newey-West t-statistics of -4.92. The IVOL effect is observed in three out of four Nordic constituents as Finland with -1.917, Sweden with -2.177, and Denmark with -1.671 are all significant with robust absolute t-values between 4.45 and 6.44. Statistically, the most significant relationship between lagged idiosyncratic volatility and future average excess returns is observed in Sweden. In the Norwegian market, the effect is not observed, despite soft evidence from the IVOL coefficient's negative front sign and t-value of -1.63. Size is observed in the Nordics with significant negative coefficients -0.108, in line with the earlier finding -0.087 for Europe in Ang et al (2009). So is the book-to-market (BE/ME) with positive significant coefficients of 0.316 with a t-value 3.64. Compared to those earlier close-to-zero findings for Europe (0.010 with t-value 3.57), the lagged return over the prior six months is significant with the coefficient 1.116 and the t-value 3.44, which is, compared to earlier U.S. findings, clearly larger in the Nordic region and each Nordic country individually. Coefficients for the FF-3 factor betas are primarily insignificant, in line with earlier findings (Ang et al., 2009; Daniel and Titman, 1997).

Table 5. FMB regression: equal-weighted OLS

Table reports Fama Macbeth (1973) OLS regressions for Nordic, its constituents, and the United States. Sample period is from January 2001 to December 2017. Nordic and its constituents are expressed in EUR, U.S. in USD. LHS variable, monthly excess returns of a firm, is regressed on constant, idiosyncratic volatility *IVOL* computed from past 1-month daily returns, contemporaneous factor loadings *βMKT*, *βSMB*, *βHML* with respect to FF3 returns computed specially for each market, and firm characteristics in the beginning of month. *Size* is log market equity value of a firm in the beginning of the month, *BE/ME* is book-to-market for a firm available six months prior. *RET lagged* is the stocks return over preceding six months. *Adj R2* is time series average of adjusted R^2s from cross sectional regressions. *N* is mean number of constituent stocks over full sample period. Newey-West robust t-statistics with four lags are reported in square brackets. Panel B reports economic effect of moving from 25th to 75th volatility percentile (-1.260 x (54.80-25.88)/100 = -0.36% per month). Panel C reports descriptive statistics for the sample.

	IVOL 1-month daily estimation period, OLS					
	Nordic	Finland	Denmark	Norway	Sweden	United States
Panel A: FMB coefficients						
Constant	1.346***	1.448***	0.328	1.118***	1.567***	1.719***
	[4.32]	*[3.63]*	*[0.76]*	*[2.96]*	*[3.65]*	*[4.05]*
IVOL	-1.260***	-1.917***	-1.671***	-0.724	-2.177***	0.109
	[-4.92]	*[-5.71]*	*[-4.45]*	*[-1.63]*	*[-6.44]*	*[0.43]*
β MKT	0.505*	0.785**	0.318	0.457	0.412	-0.042
	[1.82]	*[2.59]*	*[1.25]*	*[1.35]*	*[1.27]*	*[-0.24]*
β SMB	-0.231*	-0.419**	0.273	-0.026	0.232	-0.023
	[-1.79]	*[-2.18]*	*[1.45]*	*[-0.13]*	*[1.59]*	*[-0.43]*
β HML	-0.041	0.287	-0.062	0.087	0.099	0.023
	[-0.25]	*[1.24]*	*[-0.34]*	*[0.49]*	*[0.55]*	*[0.33]*
Size	-0.108***	-0.143***	0.057	-0.099*	-0.122***	-0.161***
	[-2.82]	*[-2.74]*	*[1.27]*	*[-1.86]*	*[-2.69]*	*[-3.36]*
BE/ME	0.316***	0.320**	0.479***	0.228	0.326***	0.507***
	[3.64]	*[2.11]*	*[3.24]*	*[1.59]*	*[2.68]*	*[5.87]*
RET lagged	1.116***	1.704***	1.558***	1.310***	1.071**	-0.256
	[3.44]	*[3.89]*	*[3.47]*	*[3.29]*	*[2.10]*	*[-0.79]*
Adj R2	0.092	0.161	0.159	0.141	0.123	0.071
N	705	118	145	157	284	3619
Panel B: Idiosyncratic volatility percentiles and economic effect						
25th pctl	25.88	23.98	21.24	27.82	27.75	24.76
75th pctl	54.80	44.62	48.04	61.37	56.85	57.26
Economic effect (25th -> 75th IVOL percentile) (%)	-0.36	-0.40	-0.45	-0.24	-0.63	0.04
Panel C: Descriptive summary, time-series averages						
ERET	0.93	0.98	0.88	0.85	1.00	1.31
ME bn	1.12	1.31	1.12	0.98	1.06	4.40
ME 50th bn	0.10	0.16	0.08	0.12	0.09	0.55
BE/ME	0.86	0.81	0.99	1.00	0.73	0.70

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Interpreting the economic effect of IVOL in Nordic, shifting from the 25th idiosyncratic volatility percentile to the 75th percentile do not add risk-reward-related return for investors but delivers 0.36% less in monthly terms, as seen in Panel B. Ang et al. (2009) reported a roughly similar level of economic effect for G7 countries. While observed volatility remains relatively stable in the 25th percentile throughout the markets, magnitude varies especially in the 75th percentile, which seems to drive the economic effect. Regarding the sample composition available in Table 5 of Panel C, the monthly excess return is 0.93% on average in the Nordic region, and varies between 0.85% and 1.00%. Average market capitalisation is EUR 1 billion, whereas the median is just EUR 100 million. U.S. firms are on average roughly five times the size of Nordic firms. Book-to-market for the Nordics is 0.86, varying between the most growth, 0.73 for Sweden, and the most value, 1.00 for Norway, the difference of which could be explained by the structure of industries.

5.2 IVOL survives the size

Weighting stock returns with a priori equity market value, cross-sectional regression results show that IVOL is even strengthened, and not diminished after giving more weight to big firms. Moving from equal to value-weighted returns in the Nordics, the IVOL coefficient increases in magnitude to -1.563 (-1.260, OLS) while losing some significance with robust t-value -3.40 (-4.92). Similar development is observed for all Nordic individual markets as magnitude increases with smaller but significant t-values. In contrast to equal-weighting, IVOL is now also observed in Norway with a coefficient of -1.360 (-0.724) and a robust t-value of -2.10 (-1.63). According to past thesis work,[10] low-volatility anomaly is non-existent in Norwegian market, a finding which I can confirm with equally weighted returns. After regressing future excess returns with proper weighting, the evidence suggests that low idiosyncratic volatility provides superior performance and also valuable information in a Norwegian context. Table 6 reports Fama Macbeth (1973) regression equation (4) results computed with a value-weighting scheme using a weighted-least-squares method following equation (8), otherwise a similar procedure as described in the previous Section 5.1. WLS method weighs the variation in a regression equation with a market equity value of each cross-sectional observation.

[10] Ostnes, K. & Hafskjaer, H. (2013). The Low Volatility Puzzle: Norwegian Evidence. BI Norwegian Business School. Unpublished MSc Thesis in Finance.

Table 6. FMB regression: value-weighted WLS

Table reports Fama Macbeth (1973) WLS regressions for Nordic, its constituents, and the United States. Sample period is from January 2001 to December 2017. Nordic and its constituents are expressed in EUR, U.S. in USD. LHS variable, monthly excess returns of a firm, is regressed on constant, idiosyncratic volatility *IVOL* computed from past 1-month daily returns, contemporaneous factor loadings *βMKT*, *βSMB*, *βHML* with respect to FF3 returns computed specially for each market, and firm characteristics in the beginning of month. *Size* is log market equity value of the firm in the beginning of the month, *BE/ME* is book-to-market for the firm available six months prior. *RET lagged* is the stocks return over preceding six months. *Adj R2* is time series average of adjusted R^2s from cross sectional regressions. *N* denotes mean number of constituent stocks over full sample period. Newey-West robust t-statistics with four lags are reported in square brackets. Panel B reports economic effect of moving from 25th to 75th volatility percentile (-1.563 x (54.80-25.88)/100 = -0.45% per month).

	IVOL 1-month estimation period, daily returns, WLS					
	Nordic	Finland	Denmark	Norway	Sweden	United States
Panel A: FMB coefficients						
Constant	2.792***	2.563***	2.566***	2.076***	3.278***	2.119***
	[4.77]	*[3.07]*	*[3.65]*	*[2.79]*	*[4.18]*	*[4.08]*
IVOL	-1.563***	-2.008**	-2.816***	-1.360**	-2.284***	-0.030
	[-3.40]	*[-2.55]*	*[-3.18]*	*[-2.10]*	*[-4.06]*	*[-0.07]*
β MKT	-0.399	-0.565	-0.161	0.513	0.019	-0.523**
	[-0.98]	*[-1.12]*	*[-0.44]*	*[1.20]*	*[0.04]*	*[-2.06]*
β SMB	-0.211	-0.066	0.122	-0.458	-0.292	-0.039
	[-1.07]	*[-0.22]*	*[0.45]*	*[-1.59]*	*[-1.43]*	*[-0.42]*
β HML	0.488**	0.888***	-0.090	0.209	0.291	0.072
	[2.10]	*[3.04]*	*[-0.34]*	*[0.95]*	*[1.09]*	*[0.79]*
Size	-0.165***	-0.129	-0.082	-0.164**	-0.264***	-0.104**
	[-2.64]	*[-1.38]*	*[-1.05]*	*[-2.20]*	*[-3.43]*	*[-2.49]*
BE/ME	0.036	-0.271	-0.390	0.018	0.191*	0.147
	[0.27]	*[-0.90]*	*[-1.54]*	*[0.09]*	*[1.71]*	*[0.88]*
RET lagged	0.246	0.301	1.350**	0.715	-0.241	0.041
	[0.55]	*[0.47]*	*[2.03]*	*[1.13]*	*[-0.41]*	*[0.11]*
Adj R2	0.218	0.425	0.374	0.271	0.273	0.163
N	705	118	145	157	284	3619
Panel B: Idiosyncratic volatility percentiles and economic effect						
25th pctl	25.88	23.98	21.24	27.82	27.75	24.76
75th pctl	54.80	44.62	48.04	61.37	56.85	57.26
Economic effect (25th -> 75th IVOL percentile)						
(%)	-0.45	-0.41	-0.75	-0.46	-0.66	-0.01

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

While not being of primary interest, the coefficients for FF-3 factor loadings diminish even further from equal-weighted results. By contrast, only the coefficient βHML becomes positive with significance from Finland. A size effect diminishes in individual countries, but increases from -0.108 to -0.165 with a robust t-value of -2.64 in the aggregated Nordic region. Book-to-market and lagged return lose basically all explanatory power after value-weighting is introduced. Economic effect increases in all Nordic markets, delivering 0.45% (0.36% OLS) in monthly terms for the aggregated Nordic region, ranging between 0.41% and 0.75% (0.24% and 0.63%) for Nordic constituents. An increase in economic effect is fully driven by larger IVOL coefficients since equal volatility percentiles are used for both weighting schemes. Larger adjusted R2 values are in range of 0.218 and 0.425 (0.092 and 0.161), indicating that the overall model fit is better with a value-weighted setting. Therefore, smaller stocks are more responsible for that variation which do not contain such valuable information with regard to the relationship between IVOL and future excess returns.

As a conclusion, according to value-weighted results, IVOL coefficients higher in magnitude, remaining significant, suggest that the effect is actually stronger among big firms. Typically, literature claims that such anomalies are often more evident within smaller stocks. Results from cross-section provide opposite evidence among Nordic-aggregated and individual markets, magnitude of IVOL effect gets larger with more weight given to large stocks. Ang et al. (2009) finds similar evidence in their international sample, value-weighted IVOL with respect to World FF-3 is -0.893 (-0.668, ew) with robust t-value -3.17 (-2.33). However, large stocks' dominance does not explicitly mean anything regarding IVOL in small stocks. To analyse further, I examine double-sort portfolios in the following sections and provide excess returns and alphas for small and large stocks individually.

5.3 Controversy in the United States

Surprisingly, I report controversial results for the United States. Past idiosyncratic volatility predicts no future excess returns in the U.S. with equal-weighting reported in Table 5, nor value-weighting reported in Table 6 over the sample period of 2001-2017. However, U.S.-related results together lead to three key conclusions: results are consistent with prior literature as IVOL is observed in the U.S. for the earlier sample period from 1980-2003, it has time varying characteristics as significant findings seem to require correct time periods to be measured, and the methodology for the paper is most likely implemented correctly.

Furthermore, even though it is not unprecedented to see such anomalies fading out, I do not attempt to answer why that may be happening. The work is restricted to reporting IVOL existence within the Nordics, comparing the results against U.S. and therefore showing the current performance of the IVOL anomaly among multiple markets. Detailed analysis on mechanisms behind the anomaly itself is left for future research. Hou and Loh (2016) report similar to Ang et al (2006, 2009) findings on IVOL using CRSP dataset and covering the period of 1963-2012. Extensive analysis is provided, for example, by using subsamples for multiple characteristics such as low analyst coverage or bad credit ratings, however, they do not report anything on different time periods. As my findings suggest that the IVOL puzzle, or the anomaly, is not existent in the United States during the main sample period of 2001-2017, I extend my work to cover a similar time period with Ang et al (2009) and confirm their findings by reporting coefficients and significances in a similar direction in the Fama Macbeth cross-section over their sample period of 1980-2003. The extension is provided as a proof of methodology implementation, results from the extension are reported in the appendix.

Regarding the U.S. sample from 2001 to 2017, IVOL remains insignificant for the whole period with value-weighted coefficient -0.030 (0.109, equal-weighted) and robust absolute t-value 0.07 (0.43). The finding is contrary to Ang et al. (2006, 2009) as they report highly significant IVOL coefficient -2.243 (-2.014) with an absolute t-value of 7.00 (6.67). Contrary results could be explained by multiple reasons, such as changes in institutional money managers' benchmarking, retail investor better exposure to arbitrage opportunities, or increased systematic trading. Also, a different dataset used for the analysis would explain contradictory findings as earlier U.S. evidence is based on CRSP data, whereas I use Datastream market data. In a value-weighted context, Ang et al. (2009) use idiosyncratic volatility estimation with respect to World FF-3 factor returns for their analysis, and do not provide comprehensive sensitivity measures for IVOL estimated with respect to regional or local FF-3. They report large co-movements between international and U.S. idiosyncratic portfolio returns. The finding which intuitively seem to diminish in my results with insignificant IVOL coefficient in the U.S. versus significant IVOL in the Nordic region.

Nordic and U.S. FF-3 factor loadings have similar front signs without large deviations in significance in a value-weighted regression setting. With value-weighting, SMB -0.039 (-0.086 in Ang et al, 2009) and HML 0.072 (-0.041) loadings remain insignificant (insignificant), the book-to-market coefficient is 0.147 with t-value 0.88 (0.241, t-value 3.20) and lagged return coefficient insignificant close to zero (equally insignificant). The FMB regression's overall fit is increased with adjusted R^2 at 0.163 in my sample whereas Ang et al. report 0.053 adjusted R^2 for their earlier U.S. sample. Size is at par level with a significant coefficient of -0.104 (-0.067). According to a recent paper from Alquist et al. (2018), size should not have a role, something which I cannot confirm with observed significant size coefficients. I do not say they are wrong, but rather pinpoint their comments on weak size effect, well-cleaned CRSP datasets, and recently refined delisting returns for CRSP, which portrayed small stocks too positively.

5.4 IVOL survives the quality

Among the Nordic-aggregated market, past idiosyncratic volatility continues predicting future excess returns after control for quality is introduced. I add the firm-level quality measure QMJ (Asness et al, 2014) into the Fama Macbeth regression equation (3) and, using equal-weighting report coefficients in Table 7. The QMJ score is data-intensive and therefore analysis is restricted to a Nordic-aggregated market and subsample of large cap stocks, defined as firms with market cap exceeding EUR 1 billion[11].

IVOL is strong within Nordic large cap, especially during the *post-crisis* environment of decreasing volatilities, where larger significant coefficients of IVOL dominates over quality. In contrast, quality outruns IVOL in predicting future excess returns among all Nordic stocks. Idiosyncratic volatility and quality, both remain significant and large. First two columns in Table 7 incorporates a full sample period, first reporting the full Nordic sample with 477 stocks and then Nordic large cap, on average, 87 stocks, consisting of stocks with required data available for QMJ calculation. For *post crisis*, seen in the third and fourth columns, the number of constituents increase up to 516 and 104 respectively. Multistage QMJ construction with extensive data requirements brings the full sample size down from 705 stocks (Table 5). For a comparison, the largest quintile in the Nordics is, on average, 131 stocks over a full sample period, indicating that a static EUR 1 billion large-cap filter poses additional cut to the sample and weights the recent past more than the beginning of the sample. For the full sample period, the Nordic IVOL coefficient remains significant at -0.826 (-1.260 before QMJ in Table 5) with some of its explanatory power lost with robust t-value at -3.01 (-4.92). Most interestingly, in addition, that IVOL losing its explanatory power, QMJ coefficient is highly significant at 1.698 with t-value 9.53.

[11] NASDAQ: the Nordic Large Cap segment includes companies with a market capitalisation equivalent to EUR 1 billion or more. Available at https://business.nasdaq.com/Docs/INET-Nordic-Market-Model.pdf

Table 7. FMB regression: QMJ introduced

Table reports Fama Macbeth (1973) WLS regressions for aggregated Nordic. *Full sample* is from January 2001 to December 2017 and *Post crisis* from July 2009 to December 2017. All values are expressed in EUR. *Nordic Large cap* is defined as firms with market capitalization above EUR 1 billion. LHS variable, monthly excess returns of a firm, is regressed on constant, idiosyncratic volatility *IVOL* computed from past 1-month daily returns, *QMJ* is beginning of month quality measure from Asness et al (2014), contemporaneous factor loadings *βMKT, βSMB, βHML* with respect to 3-FF of Nordic aggregated market, and firm characteristics in the beginning of month. *Size* is log market equity value of the firm in the beginning of the month, *BE/ME* is book-to-market for the firm available six months prior. *RET lagged* is the stocks return over preceding six months. *Adj R2* is time series average of adjusted R^2s from cross sectional regressions. *N* denotes mean number of constituent stocks over sample period. Newey-West robust t-statistics with four lags in square brackets. Panel B reports time series means: annualised volatility percentiles, monthly excess return, market capitalization in EUR billion, median for market cap, and book-to-market.

	Equal-weighted OLS			
	Full sample		Post crisis	
	Nordic	Large Cap	Nordic	Large Cap
Panel A: FMB coefficients				
Constant	0.949**	2.634***	1.507***	4.333***
	[2.57]	*[3.16]*	*[2.88]*	*[4.61]*
IVOL	-0.826***	-1.469**	-0.872**	-2.370***
	[-3.01]	*[-2.41]*	*[-2.52]*	*[-2.84]*
QMJ	1.698***	0.742***	1.545***	0.533*
	[9.53]	*[3.00]*	*[5.94]*	*[1.79]*
β MKT	0.518*	-0.302	0.363	-0.034
	[1.72]	*[-0.81]*	*[1.45]*	*[-0.11]*
β SMB	-0.238*	-0.255	-0.219*	-0.560***
	[-1.84]	*[-1.27]*	*[-1.67]*	*[-2.68]*
β HML	0.030	0.524**	0.104	0.496***
	[0.17]	*[2.18]*	*[0.71]*	*[2.94]*
Size	-0.123***	-0.172**	-0.076	-0.272***
	[-2.89]	*[-2.15]*	*[-1.47]*	*[-3.03]*
BE/ME	0.560***	0.149	0.090	-0.030
	[4.74]	*[0.69]*	*[0.92]*	*[-0.14]*
RET lagged	1.047***	0.707	1.194***	1.091*
	[2.98]	*[1.25]*	*[3.12]*	*[1.84]*
Adj R2	0.107	0.162	0.092	0.147
N	477	87	516	104
Panel B: Descriptive statistics				
IVOL 25th pctl	27.54	22.78	25.85	19.22
IVOL 75th pctl	55.12	36.69	52.81	32.10
ERET	0.92	0.93	1.17	1.40
ME bn	1.27	6.30	1.44	6.29
ME 50^{th} bn	0.12	2.78	0.14	2.87
BE/ME	0.78	0.60	0.82	0.60

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Post crisis period remains more or less at par level for all Nordic stocks. Large cap stocks are the opposite, leaving only a fraction for QMJ with coefficient 0.742 and a robust t-value 3.00 for the full sample period, whereas IVOL is clearly large at -1.469 and a robust t-value -2.41. *Post crisis* IVOL among large cap increases in magnitude with a large significant coefficient of -2.370 with t-value -2.84 while QMJ significance continues to decrease.

To conclude on idiosyncratic volatility and quality, Nordic large cap takes another direction by delivering large significant coefficients for IVOL after control for quality is introduced. Among all stocks, IVOL continues explaining the future stock returns, but does so with smaller coefficient. IVOL anomaly survives the control for quality. These findings indicate that investors require some level of quality, which seem to be on offered by large Nordic companies, and after the quality requirement is fulfilled, other factors matter. Indication is supported by the observation among all Nordic stocks, sample with small stock domination, investors demand quality, the quality is the first priority with unparalleled future return predictability. Until this far, IVOL succeeds extensive individual stock level analysis as it survives controls for size and quality. Therefore, preliminary answer for the broad question of the thesis topic would take a form: large cap stocks with good, or even just decent, quality is the rationale for success of low-volatility investing in Nordic market. In the following sections, I examine holding period returns by constructing quintile portfolios sorted on idiosyncratic volatility, then each portfolio is further sorted on size and finally on quality. To ensure robustness, sensitivity to multiple IVOL estimation periods is examined, as well as full sample is divided in three sub periods to see how performance evolves over time.

5.5 Portfolio returns

Among Nordic stock market, total volatility offers reward from taking risk as the long-short portfolio comes with large negative alpha and excess return with equal-weighted portfolios. Significance is diminished with value-weighted returns, suggesting that previous realized total volatility predicts correctly future returns especially among smaller stocks. The United States continues with contrary performance, as based on total volatility, anomaly of not getting rewarded from taking higher risk is observed only among value-weighted portfolios, suggesting that big stocks drive the anomaly. With equal-weighting, U.S. follows the theory and offer no free lunch. Idiosyncratic volatility anomaly exists in Nordics and delivers positive excess return of monthly 0.8% with equal alpha for self-financed long-short portfolio. *Pre-*

crisis and during *financial crisis* IVOL performance varies while *post-crisis* shows 1.0% monthly excess return with significant FF-3 alpha. Regarding to size, IVOL is even larger with 1.5% monthly excess return among *medium* sized Nordic stocks, *big* stocks being no shy with 1.2% return and 1.4% alpha. For small stocks, no significant numbers to report. Regarding to quality, IVOL is strongest among *junk* stocks, *neutral* stocks right behind *junk* stocks, but highest *quality* stocks post no significant IVOL returns. Actually, all quality stocks seem to perform at par level, no matter of relative idiosyncratic volatility.

Holding period excess returns and alphas for multiple portfolios are reported and constructed by sorting stocks on total volatility TVOL, idiosyncratic volatility IVOL, IVOL and size, and finally IVOL and quality. Portfolios are constructed to understand the performance of volatility return prediction in detail. For long-short portfolio long and short legs are arranged so that portfolio P1 with lowest volatility goes long and portfolio P5 short. Reported P1-P5 portfolio performance is therefore positive in case volatility anomaly is observed, which indicates that past low-volatility portfolio outperforms portfolio of high-volatility stocks. First, the stocks are divided in quintile portfolios based on total volatility TVOL computed using past month daily excess returns. Second, I divide stocks based on past month idiosyncratic volatility, apply into quintile portfolios, which then are further double sorted on market capitalization. Finally, IVOL portfolios are double sorted on quality. Holding period returns are examined by computing excess returns and alphas with respect to FF-3 risk factors for each portfolio.

If the low-volatility strategy is able to reach any of unexploited excess returns in the markets, we would like to observe positive alphas and excess returns from long-short portfolios. Following the theory, as being long with high-volatility and short with low-volatility portfolio, long-short portfolio should provide positive returns. Taking higher risk should be rewarded with higher returns. However, in portfolio construction, I do inverse to what theory suggests, portfolio P1 with lowest volatilities is long and P5 with highest volatilities short, explicitly expressed as P1-P5, where positive P1 indicates being long and respectively –P5 indicates being short. Positive return from long-short portfolio therefore reveals anomaly existence, investors are rewarded from taking lower, not higher, risk as with such portfolio construction P1 is expected to outperform portfolio P5. Idiosyncratic volatility is estimated from prior month daily returns with respect to FF-3 computed for each market. Following Bali and Cakici (2008), equal-weighted returns are used, as they find insignificant and often positive alpha for P5-P1 (not P1-P5 as seen here) strategy among their full U.S. sample. All volatility portfolios are refreshed in the beginning of each month. In addition to long-only portfolios, holding period return is calculated for theoretically self-financed long-short portfolios. In addition, in the section for robustness checks, significance is examined in Fama Macbeth (1973) cross-section with multiple IVOL estimation periods using daily and also monthly returns.

5.5.1 Portfolios sorted on TVOL

Rationale for total volatility portfolios is that either both, one of them, or neither of the components, systematic and idiosyncratic parts of past realized volatility predicts future returns. Table 8 reports total volatility portfolio holding period returns and alphas, first thing to look at is exactly opposite performance of long-short TVOL portfolios for equal-weighted Nordic and value-weighted U.S. In the first two columns we observe significant monthly alpha 1.993% with high t-value 6.98 for Nordic long-short TVOL portfolio with massive 2.572% excess return. Portfolio P5 with highest volatile stocks dominates long-short portfolio with even larger coefficient and significance. In the United States, smaller but otherwise similar coefficients are reported with equal-weighted portfolio returns. Comparing to value-weighted returns, weighting more to larger stocks, Nordic long-short TVOL portfolio alpha diminishes to zero and low-volatility dominates in portfolio P1 with 0.529% alpha and robust t-value 3.15.

Most interestingly, U.S. long-short TVOL portfolio reports alpha of -1.751% with t-value -4.01 and insignificant mean excess return. Among Nordic, total volatility predicts especially well future returns for small stocks, supported by diminishing large alpha and excess return when moving from equal- to value-weighted examination. U.S. evidence is interesting as value-weighted portfolios provides monotonically decreasing alphas, highly significant extreme portfolios P1 and P5, without any excess return from long-short portfolio. Such a puzzling behaviour would occur for example if low- and high-volatility stocks have diverse time varying characteristics and returns are miss-aligned in terms of long-short portfolio performance examination. However, investors require returns; instead of getting alpha, they require excess returns as the first priority. Total volatility portfolios provide either no significant excess return within the U.S. or high returns with high risk in the Nordics, I conclude that the idiosyncratic volatility is more interesting as the TVOL portfolios provide somewhat puzzling performance with no observable logic pattern in portfolio alphas. Next, the idiosyncratic component of the volatility is measured in Section 5.5.2 as the firm-specific part of the volatility is examined further. The IVOL portfolio holding period return performance is covered in the next sections.

Table 8. Holding period returns: TVOL

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns for each portfolio. Portfolios are constructed sorting stocks on past one-month total volatility. Portfolio *P1 (P5)* is the stocks with lowest (highest) past total volatility TVOL. Monthly excess returns, expressed in EUR for all Nordics and USD for the United States, are regressed on constant, MKT, SMB, and HML. Equal- or value-weighting refers to portfolio weighting scheme. Robust t-statistics reported in square brackets. *Full sample* is sample period from 2001/01 to 2017/12.

	Sort on σ(t-1, t), full sample			
	Equal-weighted		Value-weighted	
	Nordic	United States	Nordic	United States
Panel A: FF3 alphas				
P1 Low	-0.039	0.570***	0.529***	0.596***
	[-0.43]	*[6.49]*	*[3.15]*	*[7.15]*
P2	-0.166*	0.274***	0.449***	0.337***
	[-1.79]	*[3.49]*	*[3.03]*	*[3.98]*
P3	-0.210**	0.100	0.318*	-0.003
	[-2.46]	*[1.22]*	*[1.88]*	*[-0.02]*
P4	-0.245**	-0.139	0.228	-0.546***
	[-2.31]	*[-1.20]*	*[0.96]*	*[-3.29]*
P5 High	1.954***	1.702***	0.478	-1.155***
	[8.23]	*[4.95]*	*[0.93]*	*[-3.04]*
P1-P5	-1.993***	-1.132***	0.051	1.751***
	[-6.98]	*[-3.08]*	*[0.09]*	*[4.01]*
Panel B: Raw average excess returns				
P1-P5	-2.572***	-2.085***	-0.452	0.819
	[-3.60]	*[-3.00]*	*[-0.49]*	*[1.09]*

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

5.5.2 Portfolios sorted on IVOL

Table 9 reports holding period excess returns and alphas for full sample periods using equal-weighted quintile portfolios sorted on idiosyncratic volatility, which is estimated over the past one-month period by using daily returns. Self-financing long-short portfolio is constructed with low-volatility stocks (P1) in the long leg and high-volatility (P5) in short leg. Similarly, Table 10 to Table 12 report findings on sub samples *pre-crisis*, *financial crisis*, *post-crisis*. Figure 2 illustrates long-short portfolio's cumulative performance relative to the value-weighted market portfolio. One euro invested in the IVOL portfolio comes out four times the initial investment at the end of the sample period.

Firstly, the aggregated Nordic long-short portfolio on average shows significant alpha 0.892% with robust t-statistics at 3.55. Excess return for the long-short portfolio P1-P5 is 0.806% with a t-value of 2.15 per month. Portfolio P5 with the highest IVOL stocks has negative alpha but independently remains insignificant. By contrast, portfolio P1 with stocks with the lowest IVOL is highly significant and drives long/short portfolios' performance with monthly alpha of 0.539% and t-value of 4.98. Verifying the observation, portfolio alphas form a monotonically decreasing pattern when moving from portfolio P1 to P5 in the aggregated Nordic region. The market has no direct performance benchmarks available from prior literature, but results seem logical, aggregated Nordic coefficients follow the performance of individual constituent markets and are supported by the findings from FMB cross-sectional regressions represented in previous sections. For Europe, a sample of 1980-2003, Ang et al. (2009) report a value-weighted alpha of -0.551 (t-value -2.19) and excess return -0.412 (-1.50) for inverse P5-P1 IVOL long/short portfolio.

Secondly, all Nordic markets individually show similar pattern, two of the lowest volatility portfolios P1 and P2 deliver significant alpha in range 0.5-0.6% per month. High-volatility portfolios report no significant alpha. While Finland, Denmark, and Sweden have similar portfolio alpha structure, Norway is passive in extreme portfolios as in short leg P1 remains insignificant but then portfolio P2 performs similarly to other Nordic countries. Again, monotonically decreasing pattern is observed in alphas when moving from portfolio P1 to P5 among Nordic individual countries. Sweden has the largest excess returns 1.002% per month, other countries varies between statistically insignificant 0.084% for Norway and significant 0.731% for Finland. Stockholm School of Economics offers benchmark for Swedish findings, Elvelin and Hage (2015) report in their MSc thesis, in similar setting, that equal-weighted IVOL long-short portfolio has FF-3 alpha

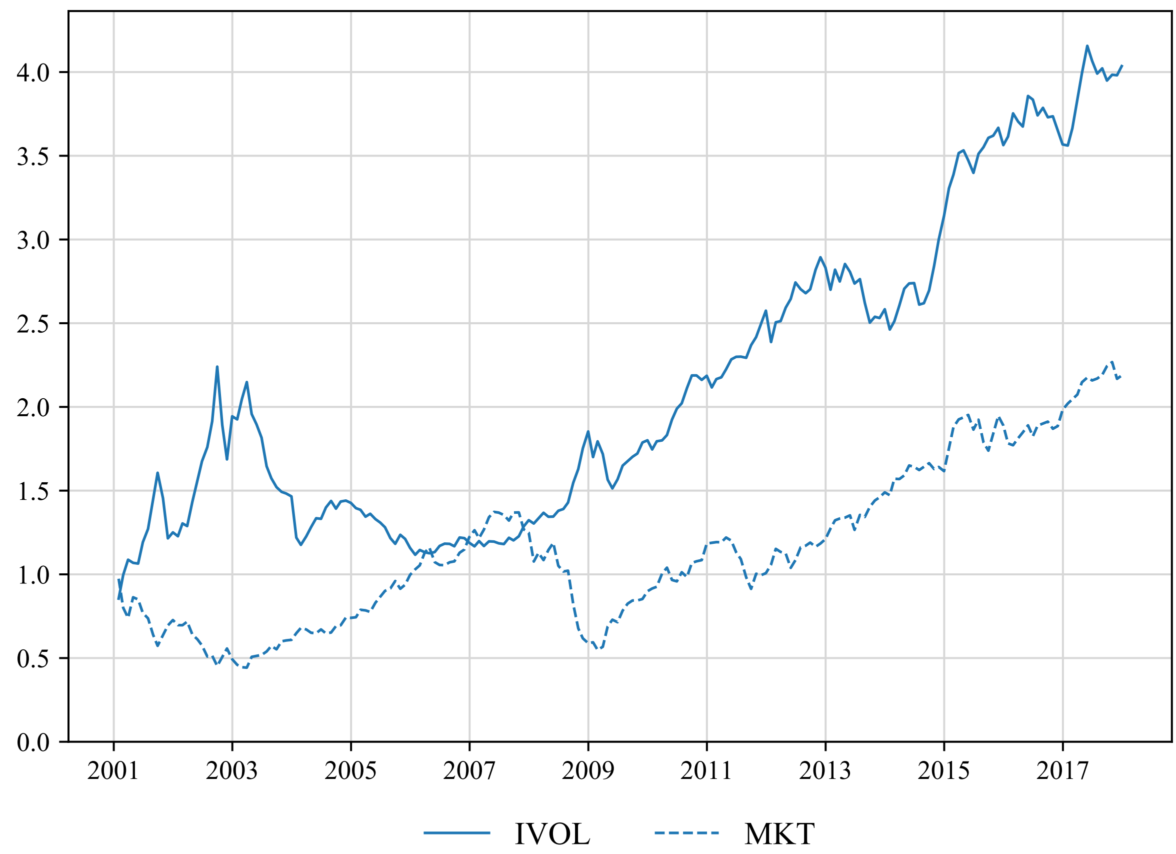

Figure 2. IVOL long short portfolio returns in Nordic
Figure illustrates cumulative excess return of one euro invested in equal-weighted IVOL long short portfolio. Low idiosyncratic volatility stocks portfolio P1 is in long leg and portfolio P5 with high idiosyncratic volatility stocks in short leg. IVOL is computed using all Nordic stocks available for idiosyncratic volatility estimation over previous one-month daily returns. *MKT* refers to market portfolio constructed following Fama French (1993), including all Nordic stocks in the sample.

of monthly 0.16% with t-value 0.30 and mean excess return of -0.10% for sample period 1994/7 - 2013/12[12]. They do not provide FMB regression results for individual stock level analysis. However, their sample includes stocks from main exchange but also from minor exchanges, like First North or Aktietorget, indicating that they have even larger domination from small stocks in their sample, stocks with insufficient shorting ability. Thirdly, even though returns are not directly comparable between Nordics (EUR) and U.S. (USD), monotonically diminishing pattern is non-existent for U.S. portfolios, neither negative alphas nor excess returns are observable for high-volatility portfolios. Therefore, long-short portfolio is insignificant with alpha and excess return with positive front signs.

[12] Elvelin, A. & Hage, U. (2015). Idiosyncratic Volatility and Risk-Adjusted Returns: Evidence from the Swedish Stock Market. Stockholm School of Economics. Unpublished MSc Thesis in Finance.

Table 9. Holding period returns: IVOL

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns (%) for each portfolio. Portfolios are constructed sorting stocks on idiosyncratic volatility. Portfolio *P1 (P5)* is the stocks with lowest (highest) past idiosyncratic volatility. P1-P5 is long-short portfolio long with lowest idiosyncratic volatility stocks and short with stocks with high idiosyncratic volatility. Monthly IVOL portfolio excess returns as LHS variable, expressed in EUR currency for all Nordics and in USD for the United States, are regressed on constant, MKT, SMB, and HML. All IVOL portfolios are equal-weighted. Robust Newey-West t-statistics with four lags reported in square brackets. *Full sample* includes all available Nordic stocks for portfolio formation from 2001/01 to 2017/12, subsample *pre-crisis* refers to 2001/01-2007/09, *financial crisis* refers to 2007/09-2009/06, and *post-crisis* is the period of 2009/07-2017/12. In Panel B, *ERET* is mean monthly return over risk-free rate, *βSMB* and *βHML* are risk factor loadings for P1-P5 long-short portfolio. Panel C report arithmetic mean returns (%) for sample and risk factor portfolios. Sample consists of stocks with required variable data available for IVOL estimation. FF-3 risk factor returns (%) are computed individually for each market and are expressed in currencies similar to IVOL portfolios, Nordics in EUR and U.S. in USD.

	Sort on $\sigma_i(t-1, t)$, equal-weighted, full sample					
	Nordic	Denmark	Finland	Norway	Sweden	United States
Panel A: FF-3 alphas						
P1 Low	0.539***	0.623***	0.600***	0.239	0.572***	0.519***
	[4.98]	*[3.93]*	*[3.35]*	*[1.64]*	*[4.39]*	*[6.38]*
P2	0.466***	0.562***	0.442***	0.606***	0.619***	0.355***
	[5.00]	*[3.58]*	*[2.98]*	*[3.94]*	*[5.41]*	*[4.58]*
P3	0.369***	0.226	0.306*	0.392**	0.382***	0.348***
	[4.17]	*[1.36]*	*[1.78]*	*[2.00]*	*[2.77]*	*[4.02]*
P4	0.024	0.323**	0.087	-0.190	-0.010	0.399***
	[0.21]	*[2.00]*	*[0.54]*	*[-0.77]*	*[-0.06]*	*[2.98]*
P5 High	-0.353*	-0.404	-0.235	-0.049	-0.180	0.751***
	[-1.82]	*[-1.65]*	*[-1.32]*	*[-0.15]*	*[-0.68]*	*[2.83]*
P1-P5	0.892***	1.027***	0.835***	0.288	0.752**	-0.232
	[3.55]	*[3.10]*	*[3.10]*	*[0.71]*	*[2.51]*	*[-0.84]*
Panel B: Excess returns and risk factor loadings						
P1-P5 ERET	0.806**	0.719**	0.731**	0.084	1.002**	-0.713
	[2.15]	*[2.18]*	*[2.47]*	*[0.14]*	*[2.59]*	*[-1.61]*
P1-P5 βSMB	-0.775***	-0.373***	-0.438***	-0.939***	-0.749***	-0.903***
	[-6.00]	*[-3.06]*	*[-3.73]*	*[-5.61]*	*[-6.82]*	*[-5.53]*
P1-P5 βHML	0.342***	-0.179**	0.070	0.006	0.552***	0.584***
	[3.65]	*[-2.09]*	*[0.79]*	*[0.05]*	*[4.64]*	*[2.69]*
Panel C: Sample returns						
Sample mean	0.663	0.957	0.410	0.906	0.713	0.654
FF-3 MKT	0.553	0.846	0.297	0.798	0.604	0.545
FF-3 SMB	-0.032	-0.394	0.116	-0.351	0.026	0.302
FF-3 HML	0.526	0.134	0.826	0.328	0.792	0.278

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Furthermore, extreme portfolios P1 and P5 report more or less equal behaviour with on par monthly alphas 0.519% and 0.755%. Middle portfolios P2-P4 are significant with 0.347-0.398% monthly alphas. Long-short portfolios among all markets are consistent with significantly negative size exposures with betas between -0.373 and -0.939. Nordic value exposure is positive with HML beta of 0.342, otherwise value exposures varies slightly. Factor loadings indicates that Nordic IVOL is long large stocks, and short small stocks, which is supported earlier FMB findings on strengthening coefficients within Nordic Large Cap. Also, Nordic IVOL is long value stocks and short growth stocks. To conclude, self-financed Nordic IVOL long-short portfolio returns are combination of extreme portfolios, and on average the portfolio is most exposed to larger undervalued firms with book values relatively closer to market values.

5.5.3 Portfolios sorted on IVOL: pre-crisis, financial crisis, and post-crisis

IVOL long-short portfolio performance is puzzling or non-existent before and during the crisis, after the crisis portfolio delivers significant and large alpha and excess returns. Table 8 reports alphas and excess returns for *pre-crisis* sub period of January 2001 to September 2007. In contrast to full period, Nordic long-short portfolio is statistically insignificant with alpha 0.646%, portfolio P1 with lowest idiosyncratic volatility is significant with alpha 0.561% and robust t-value 4.29, with monotonically decreasing pattern is observable in alphas. Lowest IVOL quintile show similar to full sample performance throughout the Nordic constituents while highest IVOL quintile is more or less statistically not different from zero. The United States is again in its own category, follows the theory with negative long-short portfolio alpha -1.251% with robust t-value -2.48, driven mainly by portfolio P5 with highest IVOL stocks.

IVOL portfolios remain insignificant over period of *financial crisis* from 2007/09 to 2009/06, seen in Table 11. Alphas are mostly positive while excess returns are clearly negative, indicating negative returns for high-volatility stocks, though with insignificant t-values. Also, statistical analysis in time series context may fall in short with observations over such a short period of time. Otherwise, *financial crisis* would offer highly interesting period for detailed volatility study but possibly those methods I apply for the analysis fail to capture period specific characteristics in detail. As seen in Figure 1, volatilities increase sharply, remaining clearly elevated over the crisis. Time series average of one-month annualised total volatility for Nordic (U.S.) goes from 38.97% (42.77%) to 52.78% (66.47%), available in appendix.

Table 10. Holding period returns: IVOL pre-crisis

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns (%) for each portfolio. Portfolios are constructed sorting stocks on idiosyncratic volatility. Portfolio *P1 (P5)* is the stocks with lowest (highest) past idiosyncratic volatility. *P1-P5* refers to portfolios which is long in P1 and short in P5. Monthly excess returns, expressed in EUR for all Nordics and USD for the United States, are regressed on constant, MKT, SMB, and HML. All portfolios are equal-weighted. Robust t-statistics reported in square brackets. *Full sample* includes all available Nordic stocks for portfolio formation from 2001/01 to 2017/12, subsample *pre-crisis* refers to 2001/01-2007/09, *financial crisis* refers to 2007/09-2009/06, and *post-crisis* is the period of 2009/07-2017/12. In Panel B, excess return is mean monthly return over risk-free rate. Sample refers to stocks included in sample with required variable data available for IVOL estimation, FF-3 risk factor returns (%) are computed individually for each market and are expressed in currencies similar to IVOL portfolios, Nordics in EUR, U.S. in USD.

	Sort on $\sigma_i(t\text{-}1, t)$, equal-weighted, pre-crisis					
	Nordic	Denmark	Finland	Norway	Sweden	United States
Panel A: FF-3 alphas						
P1 Low	0.561***	0.944***	0.924***	0.451	0.550***	0.635***
	[4.29]	*[5.19]*	*[4.56]*	*[1.66]*	*[2.90]*	*[7.02]*
P2	0.280***	0.901***	0.386	0.848***	0.689***	0.527***
	[2.64]	*[4.63]*	*[1.28]*	*[3.46]*	*[4.66]*	*[6.68]*
P3	0.121	0.553**	0.450	0.534	0.480**	0.592***
	[0.63]	*[2.59]*	*[1.55]*	*[1.58]*	*[2.41]*	*[5.33]*
P4	-0.066	0.247	0.103	-0.231	0.144	0.903***
	[-0.27]	*[0.84]*	*[0.44]*	*[-0.52]*	*[0.53]*	*[4.58]*
P5 High	-0.085	-0.651	-0.336	0.566	0.390	1.886***
	[-0.26]	*[-1.47]*	*[-0.97]*	*[1.12]*	*[1.03]*	*[3.75]*
P1-P5	0.646	1.595***	1.260***	-0.115	0.161	-1.251**
	[1.65]	*[3.51]*	*[3.33]*	*[-0.18]*	*[0.37]*	*[-2.48]*
Panel B: Excess returns						
P1-P5	0.452	0.629	1.155**	-0.968	0.883	-1.379*
	[0.56]	*[1.02]*	*[2.02]*	*[-0.80]*	*[1.12]*	*[-1.73]*
Sample mean	0.820	1.157	0.571	1.645	0.753	0.509
FF-3 MKT	0.590	0.923	0.345	1.412	0.522	0.276
FF-3 SMB	0.283	0.340	0.293	-0.289	0.354	0.578
FF-3 HML	1.574	0.920	1.539	1.196	1.679	0.642

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Table 11. Holding period returns: IVOL financial crisis

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns (%) for each portfolio. Portfolios are constructed sorting stocks on idiosyncratic volatility. Portfolio *P1 (P5)* is the stocks with lowest (highest) past idiosyncratic volatility. *P1-P5* refers to portfolios which is long in P1 and short in P5. Monthly excess returns, expressed in EUR for all Nordics and USD for the United States, are regressed on constant, MKT, SMB, and HML. All portfolios are equal-weighted. Robust t-statistics reported in square brackets. *Full sample* includes all available Nordic stocks for portfolio formation from 2001/01 to 2017/12, subsample *pre-crisis* refers to 2001/01-2007/09, *financial crisis* refers to 2007/09-2009/06, and *post-crisis* is the period of 2009/07-2017/12. In Panel B, excess return is mean monthly return over risk-free rate. Sample refers to stocks included in sample with required variable data available for IVOL estimation, FF-3 risk factor returns (%) are computed individually for each market and are expressed in currencies similar to IVOL portfolios, Nordics in EUR, U.S. in USD.

	Sorted on $\sigma_i(t\text{-}1, t)$, equal-weighted, financial crisis					
	Nordic	Denmark	Finland	Norway	Sweden	United States
Panel A: FF-3 alphas						
P1 Low	0.091	-0.858	0.087	-0.531	1.479***	0.189
	[0.40]	*[-1.69]*	*[0.17]*	*[-1.11]*	*[2.93]*	*[1.58]*
P2	0.850*	-0.210	0.992	0.235	0.760	0.387***
	[1.71]	*[-0.21]*	*[1.47]*	*[0.41]*	*[1.49]*	*[2.91]*
P3	0.672**	-0.812	0.757	1.993*	1.012*	0.640
	[2.35]	*[-1.07]*	*[1.00]*	*[1.84]*	*[1.83]*	*[1.16]*
P4	0.783	-0.118	0.407	0.493	0.588	1.313
	[1.61]	*[-0.16]*	*[0.35]*	*[0.56]*	*[0.90]*	*[1.48]*
P5 High	0.008	-0.217	-0.271	-0.541	0.581	2.077
	[0.02]	*[-0.51]*	*[-0.36]*	*[-0.48]*	*[0.57]*	*[1.44]*
P1-P5	0.083	-0.640	0.358	0.010	0.897	-1.888
	[0.18]	*[-1.10]*	*[0.57]*	*[0.01]*	*[0.96]*	*[-1.23]*
Panel B: Excess returns						
P1-P5	1.374	0.384	0.670	1.801	1.714**	-0.814
	[1.16]	*[0.33]*	*[1.26]*	*[0.88]*	*[2.09]*	*[-0.38]*
Sample mean	-2.534	-2.238	-2.904	-2.581	-2.381	-1.812
FF-3 MKT	-2.642	-2.350	-3.022	-2.674	-2.486	-1.917
FF-3 SMB	-0.086	-2.022	0.495	-0.777	-0.104	0.282
FF-3 HML	-0.230	0.575	0.631	-0.346	-0.759	-0.294

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Table 12. Holding period returns: IVOL post-crisis

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns (%) for each portfolio. Portfolios are constructed sorting stocks on idiosyncratic volatility. Portfolio *P1 (P5)* is the stocks with lowest (highest) past idiosyncratic volatility. *P1-P5* refers to portfolios which is long in P1 and short in P5. Monthly excess returns, expressed in EUR for all Nordics and USD for the United States, are regressed on constant, MKT, SMB, and HML. All portfolios are equal-weighted. Robust t-statistics reported in square brackets. *Full sample* includes all available Nordic stocks for portfolio formation from 2001/01 to 2017/12, subsample *pre-crisis* refers to 2001/01-2007/09, *financial crisis* refers to 2007/09-2009/06, and *post-crisis* is the period of 2009/07-2017/12. In Panel B, excess return is mean monthly return over risk-free rate. Sample refers to stocks included in sample with required variable data available for IVOL estimation, FF-3 risk factor returns (%) are computed individually for each market and are expressed in currencies similar to IVOL portfolios, Nordics in EUR, U.S. in USD.

	Sorted on $\sigma_i(t\text{-}1, t)$, equal-weighted, post-crisis					
	Nordic	Denmark	Finland	Norway	Sweden	United States
Panel A: FF-3 alphas						
P1 Low	0.511***	0.484***	0.450**	0.366***	0.450***	0.367***
	[6.38]	*[3.06]*	*[2.22]*	*[2.63]*	*[3.03]*	*[6.29]*
P2	0.333***	0.507**	0.464***	0.610***	0.513***	0.208***
	[2.88]	*[2.36]*	*[2.85]*	*[2.97]*	*[3.69]*	*[3.49]*
P3	0.401***	0.279	0.246	0.157	0.355	0.150**
	[3.67]	*[1.07]*	*[1.34]*	*[0.73]*	*[1.70]*	*[2.23]*
P4	-0.041	0.279	0.150	-0.251	-0.198	0.038
	[-0.30]	*[1.40]*	*[0.76]*	*[-0.86]*	*[-1.04]*	*[0.31]*
P5 High	-0.359*	-0.247	-0.181	-0.207	-0.205	0.124
	[-1.70]	*[-0.95]*	*[-0.76]*	*[-0.59]*	*[-0.66]*	*[0.51]*
P1-P5	0.871***	0.731***	0.631**	0.573	0.654*	0.243
	[3.46]	*[2.61]*	*[1.98]*	*[1.51]*	*[1.78]*	*[0.86]*
Panel B: Excess returns						
P1-P5	0.969***	0.859**	0.407	0.565	0.950**	-0.163
	[3.24]	*[2.35]*	*[1.19]*	*[1.25]*	*[2.27]*	*[-0.45]*
Sample mean	1.196	1.457	0.964	1.036	1.317	1.278
FF-3 MKT	1.182	1.443	0.942	1.025	1.305	1.266
FF-3 SMB	-0.272	-0.643	-0.103	-0.312	-0.208	0.086
FF-3 HML	-0.150	-0.582	0.300	-0.222	0.407	0.108

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Post-crisis sub period is reported in Table 12 with sample period from 2009/07 to 2017/12. As indicated already by FMB regression in section 5.4, *post-crisis* is success for IVOL in Nordic aggregated market, especially among large cap stocks. Among all stocks, Nordic long-short portfolio has significant 0.871% monthly alpha with t-value 3.46 and monotonically decreasing portfolio alphas from P1 to P5. Excess return is on average at 0.969%, also with robust and significant t-value 3.24. Interestingly, only Denmark and Finland show similar to Nordic aggregated portfolios pattern with significant numbers. Other Nordic constituents' alphas remain significantly not different from zero even though they have similar front signs, align with Nordic aggregated market. Regarding to lowest IVOL portfolios, the United States behave similarly to Nordic portfolios as the portfolio P1 is highly significant with positive alpha, equally so are portfolios P2 and P3 with second and third lowest IVOL stocks. Some evidence is observed in the U.S. portfolios with the monotonic alpha pattern while remaining not significantly different from zero.

To conclude on multiple sub periods on portfolio analysis regarding to holding period returns, IVOL anomaly delivers significant and robust FF-3 alpha in Nordic aggregated market pre- and post-crisis. During financial crisis, IVOL remains insignificant but alphas in fact report positive front signs even though the market in general went south. *Post-crisis* the lowest volatility portfolios, long-only strategy, does well with significant and robust alphas and excess returns, observation which is extended throughout the Nordic markets and the United States. Even further, regarding to realistic implementation, Nordic aggregated self-financing long-short idiosyncratic volatility portfolio delivers close to 1% monthly alpha and excess return for post-crisis era. While it is reasonable to question short leg implementation among high-volatility stocks for several reasons, it is worth to notice that lowest IVOL portfolio P1 returns are at similar level to market portfolio performance and comes with relatively lower risk. Therefore, long-only analysis would be even more interesting for Nordic market with limited derivatives offering and therefore with limited access to synthetic positions. Rest of the analysis is restricted to Nordic aggregated market, IVOL relation to size and quality is examined in the following sections. Further analysis of U.S. market is left for another research attempt.

5.5.4 Portfolios double sorted on IVOL and size

Based on individual stock level findings of FMB cross-sectional coefficients, idiosyncratic volatility anomaly is strengthened when the weighting scheme is changed from equal-weighted to value-weighted returns. Figure 3 illustrates cumulative excess returns for Nordic IVOL among each size category. In terms of excess returns, one euro invested in *IVOL Medium* would have come out at the end of sample period over 14 times the original investment. *IVOL Big* would have returned close to eight times the original investment while IVOL among the smallest stocks suck up the initial investment returning less than one euro. Figure 4 illustrates portfolio alphas among each size segment and reported in Table 13 with excess returns and t-statistics. The first observation is that big firms has significant monthly alpha of 1.359% with t-value 3.77 and 1.206% excess return with robust t-value 2.70 for long-short IVOL portfolio. Among medium sized firms, significant alpha is even larger at 1.617% with mean excess return of 1.482%. For big firms, portfolio alphas and mean returns show monotonically decreasing pattern, indicating that the finding is consistent with true IVOL anomaly existence in Nordic aggregated market. Pattern illustrates the development of the effect throughout the stock universe under examination. The most extreme portfolios, P1 and P5, are used for long/short portfolio formation, however, especially among big firms, the long/short portfolio with portfolios P2 and P4 would provide close-to-similar results with robust and significant coefficients.

Primary concentration of the study remains in large firms, even though they lack a bit performance compared to medium firms as the long-short portfolio consisting big firms earns significant alpha 1.359% versus medium size long/short portfolio 1.617%. However, average size of medium portfolio stock is EUR 111 million in market equity value. Shorting restrictions and trading costs are likely to have an effect on long-short strategy in real-life trading scenario. As the section 5.5 aims to extend the analysis towards strategy implementation, it is reasonable to conclude that firm with 100 million in market equity value offers long-only potential for the strategy, if any. Detailed analysis would be required to comprehensively understand the medium sized portfolio composition as the mean size indicates nothing about its distribution metrics. In contrast, a portfolio of big firms with the largest 247 stocks are on average EUR 2.5 billion in market capitalisation, contributing to 95% of total market. For comparison, largest quintile of Nordic market includes on average of 130 firms, suggesting that the portfolio of big stocks in this research setting includes firms with market equity value well above current EUR 1 billion which is defined as market value cut for Large Cap stocks by OMX Nordic. Portfolio double sort on IVOL and size suggests that effect diminishes when moving either one of the ends, *small* portfolio is weakest, the effect is largest among *medium* sized stocks, while *big* stocks show decreasing but large and significant coefficients.

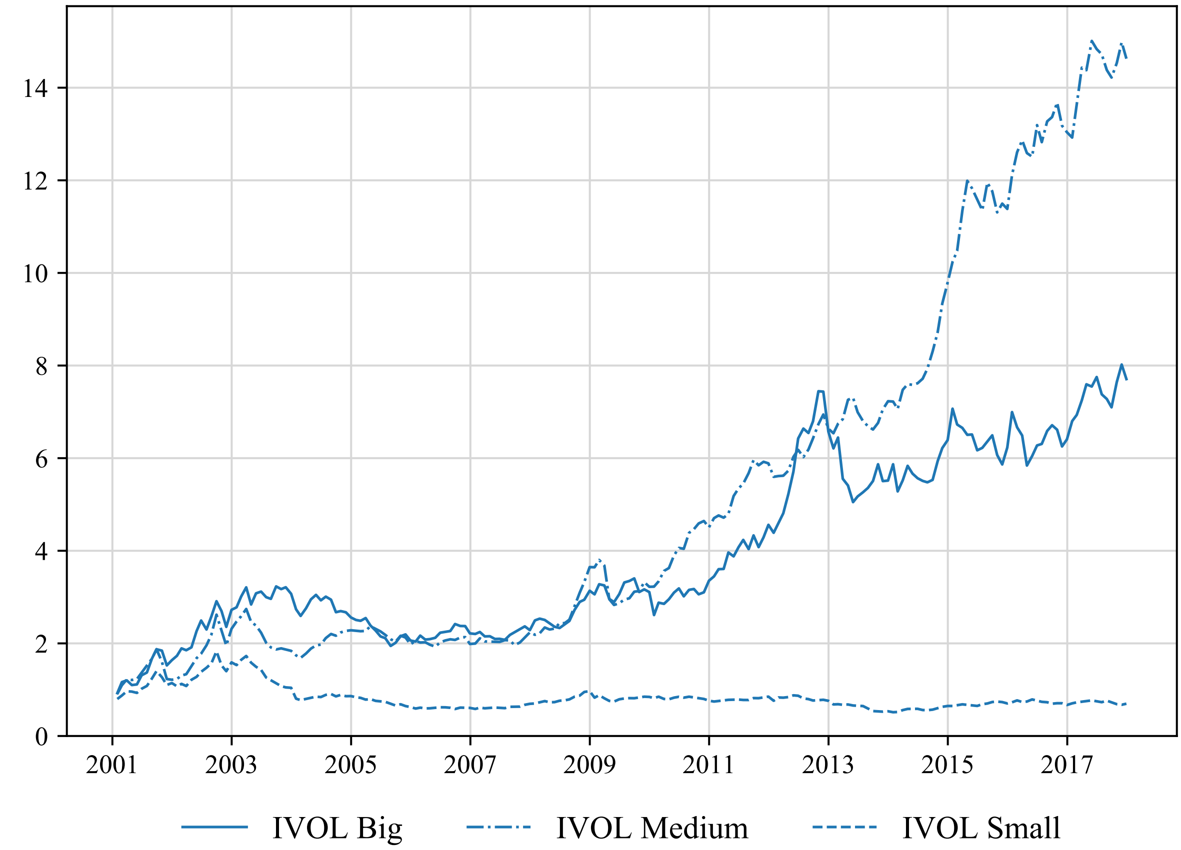

Figure 3. IVOL and SIZE long short portfolio returns in Nordic

Figure illustrates cumulative excess return of one euro invested in equal-weighted IVOL long short portfolios within three size categories. *IVOL BIG* is the IVOL long short portfolio among big stocks, *IVOL Medium* refers to portfolio of medium sized stocks, and *IVOL Small* is the portfolio of small stocks long on low idiosyncratic volatility stocks and short with high idiosyncratic volatility stocks. Portfolios are computed using all Nordic stocks available for idiosyncratic volatility estimation over previous one-month daily returns, beginning-of-month equity market value is used for size sort. Size breakpoints are 30th and 70th percentiles.

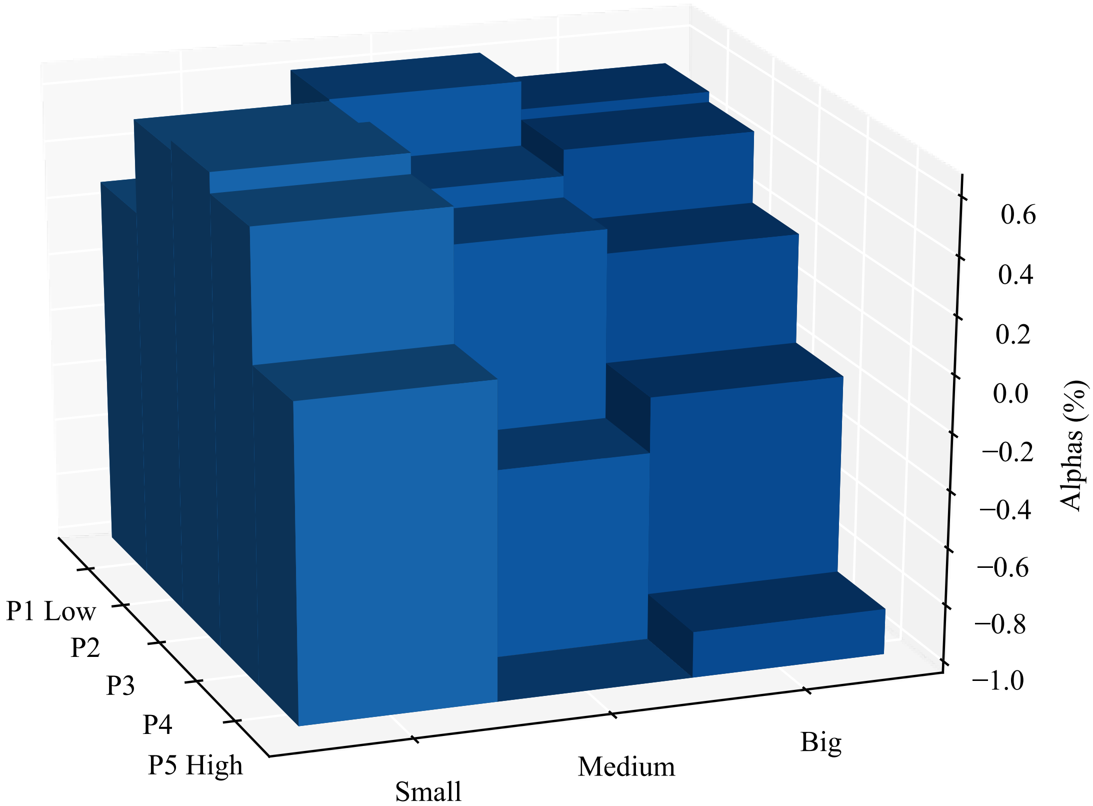

Figure 4. IVOL and SIZE portfolio alphas in Nordic
Figure illustrates monotonically decreasing pattern among medium and big alphas when moving from P1 to P5. Long-short portfolios is constructed by going long P1 and short P5, earning the spread. *Alphas* refers equal-weighted IVOL portfolio alphas with respect to FF-3, within three size categories. *P1 Low* is the portfolio with lowest IVOL stocks, highest on IVOL are in *P5 High*. *Small, Medium,* and *Big* refers to size portfolios. Stocks are divided in three by using 30th and 70th percentiles of market equity value as breakpoints. Sample period is from 2001/01 to 2017/12.

Table 13. Holding period returns: IVOL and size

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns for each portfolio. Portfolios are constructed double sorting stocks on idiosyncratic volatility and market equity value. Portfolio *P1 (P5)* is the stocks with lowest (highest) past idiosyncratic volatility. Size division is done with 30th and 70th percentiles. Monthly excess returns, expressed in EUR for all Nordics, are regressed on constant, MKT, SMB, and HML. All portfolios are equal-weighted. Robust t-statistics reported in square brackets. Sample includes on average 738 Nordic stocks for portfolio formation from 2001/01 to 2017/12. *N* refers to number of stocks. *ME mean* is mean market equity in the beginning of the month expressed in EUR million. *ME contribution* describes the share portfolio contributes to total market equity value.

	Nordic, equal-weighted								
	Small	Medium	Big	Big-Small		Small	Medium	Big	Big-Small
Panel A: FF-3 alphas					*Panel B: Excess returns*				
P1 Low	0.282*	0.611***	0.514***	0.232	P1 Low	0.752**	1.277***	1.235***	0.484**
	[1.82]	*[4.67]*	*[4.12]*	*[1.27]*		*[2.40]*	*[3.22]*	*[2.93]*	*[2.13]*
P2	0.605***	0.387***	0.478***	-0.127	P2	1.264***	1.121**	1.251***	-0.013
	[4.10]	*[3.10]*	*[3.94]*	*[-0.68]*		*[3.11]*	*[2.56]*	*[2.71]*	*[-0.06]*
P3	0.637***	0.313**	0.228*	-0.409**	P3	1.319***	1.104**	1.051**	-0.268
	[4.59]	*[2.55]*	*[1.88]*	*[-2.55]*		*[3.03]*	*[2.06]*	*[2.05]*	*[-1.24]*
P4	0.565***	-0.346**	-0.152	-0.718***	P4	1.218**	0.446	0.701	-0.517*
	[3.46]	*[-2.41]*	*[-0.94]*	*[-3.34]*		*[2.51]*	*[0.74]*	*[1.12]*	*[-1.83]*
P5 High	0.104	-1.006***	-0.845**	-0.949**	P5 High	0.774	-0.206	0.029	-0.745*
	[0.47]	*[-3.91]*	*[-2.45]*	*[-2.48]*		*[1.32]*	*[-0.29]*	*[0.04]*	*[-1.84]*
P1-P5	0.178	1.617***	1.359***	1.181***	P1-P5	-0.022	1.482***	1.206***	1.229***
	[0.58]	*[5.19]*	*[3.77]*	*[2.89]*		*[-0.05]*	*[3.22]*	*[2.70]*	*[2.97]*

Panel C: Descriptive

	N			ME			*ME contribution*		
	S	M	B	S	M	B	S	M	B
P1	34	42	71	22	116	4841	0.001	0.006	0.465
P2	27	49	71	23	117	3030	0.001	0.007	0.266
P3	37	55	56	23	114	2065	0.001	0.008	0.143
P4	56	56	35	21	110	1454	0.001	0.008	0.065
P5	91	43	14	19	100	1160	0.002	0.005	0.019

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Either smaller end of *big* portfolio's stocks dominate with large IVOL coefficients or portfolio includes stocks with particular characteristics and that specific characteristic then explains IVOL. Next, I report holding period return performance for portfolios double sorted first on idiosyncratic volatility IVOL and then on quality QMJ to examine IVOL among each quality category.

5.5.5 Portfolios double sorted on IVOL and QMJ

Figure 5 illustrates IVOL performance among each quality portfolios. *IVOL Junk* returns close to three times one invested euro over sample period and in terms of excess returns. IVOL *Neutral* follows and brings the investment back in double. By contrast, IVOL among high quality stocks deliver no positive return over sample period. To shortly conclude, IVOL is strongest in undervalued big and medium sized stocks which are either junk or neutral in quality. Detailed analysis covering alphas, excess returns and factor loadings follows below.

QMJ score, used to measure quality, is wide variable covering multiple firm characteristics and combining them in one single variable through averaging computed z-scores for *profitability*, *growth*, and *safety*. Nordic large cap would be especially interesting scope for quality double sort with relatively good past idiosyncratic volatility predictability on future returns. Nonetheless, concentrating only on large cap would cut the number of stocks in the sample so few that robustness of statistical analysis is not secured. The QMJ factor from Asness et al (2014) has some extensive requirements for data, and therefore sample is naturally restricted to fewer number of Nordic stocks, forcing to start with full Nordic stock universe, and then using 3-by-3 portfolios instead of quintile division. Table 14 reports FF-3 alphas and mean excess returns for on average 487 stocks included in sample after QMJ computation requirements, leaving mean of 54 stocks per each of 3-by-3 portfolios. Stocks are first sorted on IVOL and then divided in three QMJ portfolios. Long-short IVOL portfolio alpha is significant and positive among *junk J* and *neutral M* portfolios with monthly 0.627% (2.20) and 0.569% (2.39) coefficients. Excess returns are barely on same direction, providing 0.640% (1.79) for *junk* and 0.459% (1.45) for *neutral M* portfolios respectively.

All high-quality portfolios earn significant and robust large alphas and excess returns with equal in magnitude, alphas between 0.803% - 0.902% and excess returns 1.482% - 1.631% on average in monthly terms. *Quality Q* portfolios long-only performance is unparalleled while

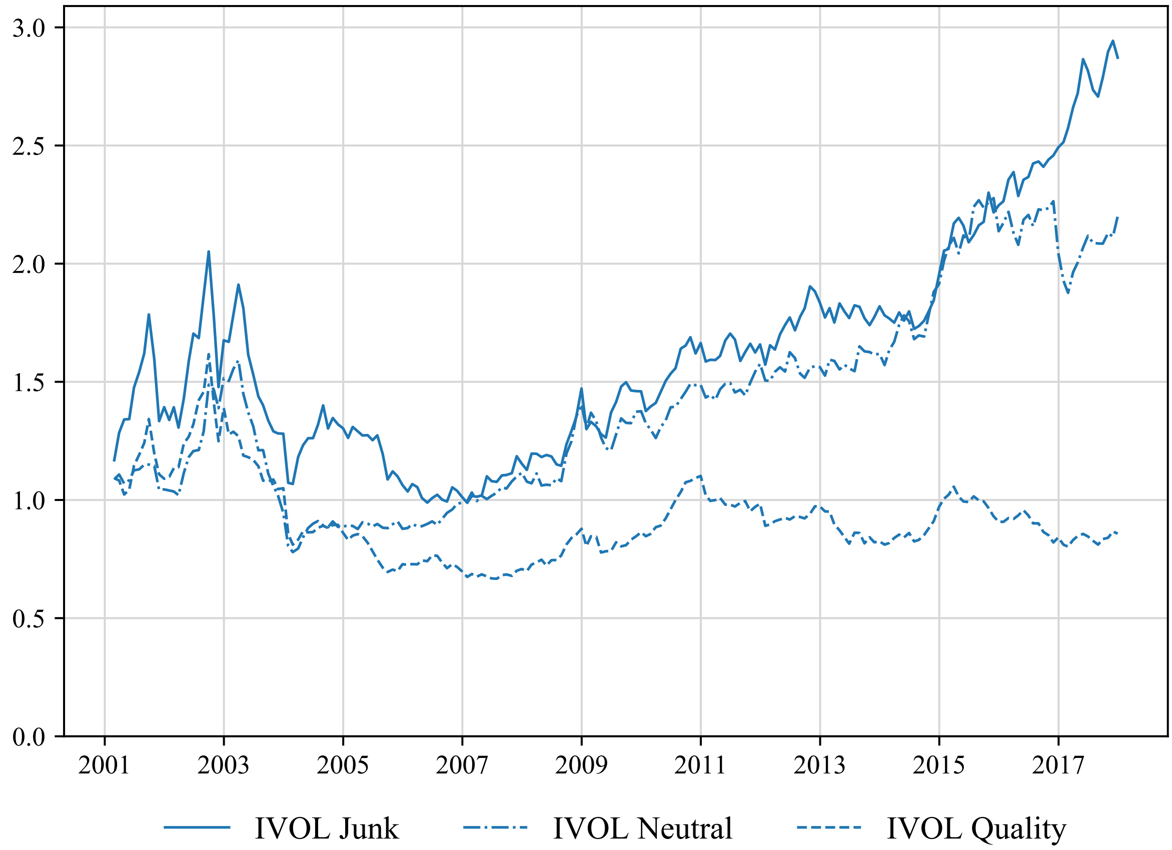

Figure 5. IVOL and QMJ long short portfolio returns in Nordic

Figure illustrates cumulative excess return of one euro invested in equal-weighted IVOL long short portfolios within three quality categories. Quality is defined following the procedure for QMJ factor from Asness et al (2014). *IVOL Junk* is the IVOL long short portfolio among junk stocks, *IVOL Neutral* refers to portfolio of stocks with neutral quality, and *IVOL Quality* is the portfolio of highest quality long on low idiosyncratic volatility stocks and short with high idiosyncratic volatility stocks. Portfolios are computed using all Nordic stocks which are available for idiosyncratic volatility estimation over previous one-month daily returns and fulfills extensive QMJ factor computation requirements. QMJ breakpoints are 30th and 70th percentiles, applied to beginning-of-month QMJ scores of firms available in cross-section.

the IVOL effect is non-existent among these portfolios. Actually, long-short IVOL QMJ portfolio even changes front sign with -0.597% alpha (t-value -2.28) and excess return of -0.638% (-2.43), indicating that in such research setting, controlling for quality in portfolio level, the theory is respected with risk-reward relation observable as the high-volatile stocks (P3) earns more than stocks with low volatility (P1). Quickly looking, portfolio specific size measures in Table 14 Panel C reveals that junk stocks are relatively small, average ME contribution of portfolios decreases consistently when moving from *Q* to *J*, and large long-only alphas are observed among biggest stocks as, in contrast, *Q* portfolios with highest *quality* have relatively large firms with doubled mean market equity value compared to *junk J* mean sizes.

Table 14. Holding period returns: IVOL and QMJ

Table reports alphas with respect to market specific FF-3 (see equation 13) and excess returns for each portfolio. Portfolios are constructed double sorting stocks on idiosyncratic volatility and then on quality. Portfolio *P1* (*P3*) is the stocks with lowest (highest) past idiosyncratic volatility. *P1-P3* is long-short portfolio, long in P1 and short in P3 portfolios. QMJ refers to long-short portfolio with high quality portfolio long and junk portfolio short. Portfolio *J* refers to junk stocks, *M* to neutral stocks, and *Q* to quality stocks. Monthly excess returns, expressed in EUR, are regressed on constant, MKT, SMB, and HML. All portfolios are equal-weighted. *P1-P3 βSMB* and *P1-P3 βHML* are risk factor loadings of long-short P1-P3 portfolio. Robust t-statistics reported in square brackets. Sample includes all available Nordic stocks for portfolio formation in the period from 2001/01 to 2017/12. *N* refers to number of stocks in each portfolio. *ME* refers to mean market equity value, *ME contribution* is the share the portfolio contributes to total market value.

	Nordic, equal-weighted								
	J	M	Q	QMJ		J	M	Q	QMJ
Panel A: FF-3 alphas and risk factor loadings					*Panel B: Excess returns*				
P1 Low	-0.310**	0.431***	0.833***	1.143***	P1 Low	0.442	1.116***	1.484***	1.042***
	[-2.05]	*[3.91]*	*[6.56]*	*[7.55]*		*[0.94]*	*[2.75]*	*[3.65]*	*[6.45]*
P2	-0.364**	0.377***	0.902***	1.266***	P2	0.521	1.121**	1.631***	1.111***
	[-2.60]	*[3.14]*	*[5.88]*	*[6.15]*		*[0.87]*	*[2.28]*	*[3.30]*	*[4.86]*
P3 High	-0.937***	-0.137	0.803***	1.740***	P3 High	-0.197	0.657	1.482**	1.679***
	[-4.24]	*[-0.74]*	*[4.61]*	*[7.23]*		*[-0.30]*	*[1.10]*	*[2.56]*	*[6.74]*
P1-P3	0.627**	0.569**	0.030	-0.597**	P1-P3	0.640*	0.459	0.002	-0.638**
	[2.20]	*[2.39]*	*[0.13]*	*[-2.28]*		*[1.79]*	*[1.45]*	*[0.01]*	*[-2.43]*
P1-P3 βSMB	-0.727***	-0.614***	-0.501***	0.225**					
	[-7.28]	*[-5.57]*	*[-5.11]*	*[2.08]*					
P1-P3 βHML	0.374***	0.108	0.272***	-0.102					
	[4.14]	*[1.60]*	*[3.67]*	*[-1.48]*					

Panel C: Descriptive

	N			*ME mean*			*ME contribution*		
	J	M	Q	J	M	Q	J	M	Q
P1	42	62	59	1670	2222	3451	0.12	0.23	0.33
P2	49	54	59	731	912	1187	0.06	0.08	0.11
P3	71	46	45	188	296	406	0.02	0.02	0.03

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

While the evidence from earlier sections suggests, size is not the dominant driver behind success of IVOL, or at least the effect is not explained by small stocks, I would point out that among highest quality *Q* portfolios only the lowest IVOL portfolio P1 is remarkable big with average firm size of EUR 3.451 billion and 33% total market equity contribution. High-IVOL portfolio with the most quality *Q* is only EUR 406 million on average firm size with 3% total market equity value contribution. Quality definitely dominates, IVOL and quality seem to have similarities in time-varying profiles, partly explaining each other, especially among big stocks, but it would be still too early to make a judgement that IVOL is overruled by quality. Factor loadings are similar to what was observed in single sort portfolios, long-short portfolio size coefficient is significantly negative, indicating that IVOL is long large firms and short on small firms. Value beta continues to suggest that IVOL, among extreme quality portfolios, is exposed to high book value stocks being long undervalued stocks and short fully valued stocks. Among *junk* stocks the short portfolio P3 dominates with larger and significant coefficient offering slight evidence that long-short portfolio returns are driven by risk factor short legs, i.e. small and overvalued firms.

To conclude on IVOL and quality, using QMJ as quality measure, idiosyncratic volatility anomaly is observed and provides consistent evidence within *junk* stocks in Nordic aggregated market. Among high *quality* stocks the IVOL anomaly is non-existent. Portfolios with highest quality, with low or high IVOL, provide unparalleled long-only performance with coefficients collectively larger than any another time-series regression examined in this study. Furthermore, I confirm that QMJ performs well in Nordic aggregated market, even though not thoroughly reported in this this paper, and succeed to divide the stocks to portfolios, with IVOL, so that unparalleled alphas and excess returns are observed among the portfolios of highest quality stocks.

5.6 Robustness

Analysis so far covers multiple examinations on IVOL anomaly, methodology is robust and similar to past literature, implementing earlier applied most demanding approaches, and after all Nordic IVOL survives the controls. The study provides clear evidence on low-volatility anomaly, past IVOL predicts future returns, and does so with significant coefficients after control for size and quality. Alphas and excess returns are observed with monotonically developing patterns for long-short portfolios examined in the study. However, as seen especially within intersection of small and medium size portfolios, medium and large stocks have the performance but at some point, below the medium/large divider, the effect starts to lose its consistency. For the future, I would examine the size sensitivity even further among the largest stocks. Even further on size, correlation coefficients, available in the Appendix, indicate that IVOL and size are positively correlated in Nordic. However, in the United States we observe negative correlation between them. Nordic Pearson correlation coefficient for IVOL and size is positive at 0.09 while Spearman coefficient is increased at 0.16. The difference in Pearson and Spearman coefficients offers slight evidence that the relation between IVOL and size may have other than linear characteristics.

The proof of methodology implementation itself is provided as an extension by replicating and confirming earlier findings of Ang et al (2009) using equal sample period with their paper. Results of the extension are reported in the appendix. Robustness is further checked in the following subsection by examining the sensitivity of idiosyncratic volatility estimation period to future return predictability.

5.6.1 Multiple idiosyncratic volatility estimation periods

Table 15 reports FMB cross-sectional individual stock level coefficients for multiple IVOL estimation periods using comparable regression setting, following *ceteris paribus*. Base case for the study is previous one-month IVOL represented in the first column. Short estimation periods are most significant among Nordics. Previous one- and six-month estimation periods using daily returns overrules longer estimation periods with larger coefficients and increased robust t-values.

In Panel A, using full sample with equal-weighted return variation in OLS regression setting, one-month estimation period shows -1.260 coefficient with t-value -4.92 while the longer six-month estimation period is at -1.356 and -4.41. Longer estimation periods, 12-months with daily, or 12- to 36- months with monthly returns, bring the coefficient down with decreasing significance. In Panel B, cross-sectional variation is weighted with WLS regression setting using a priori market values, similar pattern is reported with increased pace. After one- and six-months estimation periods the significance has diminished.

Table 15. FMB: sensitivity to volatility estimation period

Table represents results from Fama Macbeth cross-sectional regression on main sample 2001/01-2017/12. Multiple estimation periods and frequencies used for IVOL estimation. IVOL is estimated as a standard deviation measured from Fama French 3-factor regression error term for each stock. Daily and monthly returns with multiple estimation periods 1-36 months represented in the table. Nordic returns are EUR-nominated and the United States returns USD-nominated. Robust Newey-West t-statistics with four lags are represented in square brackets. OLS regression in Panel A and C indicates original equal-weighted Fama Macbeth procedure. WLS regression in Panel B is Fama Macbeth procedure extended with value-weighting with ex-ante market equity values. Panel C includes stocks above 1 billion in equity market value, replicating Nordic Large Cap stock universe.

	IVOL estimation period sensitivity					
	Daily returns			Monthly returns		
	1-month	6-month	12-month	12-month	24-month	36-month
Panel A: IVOL coefficients and t-statistics OLS, full sample						
Nordic	-1.260***	-1.356***	-1.016***	-0.730**	-1.049**	-1.252**
	[-4.92]	*[-4.41]*	*[-3.79]*	*[-1.99]*	*[-2.48]*	*[-2.47]*
United States	0.109	0.384	0.436	0.162	0.435	0.172
	[0.43]	*[1.06]*	*[0.97]*	*[0.55]*	*[1.45]*	*[0.59]*
Panel B: IVOL coefficients and t-statistics WLS, full sample						
Nordic	-1.563***	-1.632***	-0.971	0.084	0.179	0.125
	[-3.40]	*[-2.64]*	*[-1.34]*	*[0.15]*	*[0.26]*	*[0.16]*
United States	-0.030	-0.058	-0.302	-0.257	0.387	0.334
	[-0.07]	*[-0.12]*	*[-0.53]*	*[-0.66]*	*[0.94]*	*[0.86]*
Panel C: IVOL coefficients and t-statistics OLS, full sample ME minimum 1bn						
Nordic	-1.708***	-2.770***	-1.688**	-0.866	-0.519	-0.569
	[-2.96]	*[-3.46]*	*[-2.06]*	*[-1.46]*	*[-0.73]*	*[-0.71]*
United States	-0.231	-0.524	-0.530	-0.353	0.008	-0.212
	[-0.54]	*[-1.14]*	*[-0.94]*	*[-0.95]*	*[0.02]*	*[-0.63]*

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

Panel C reports equal-weighted variation among largest Nordic stocks, providing largest coefficients -2.770 with t-value -3.46 for six-month estimation period. Six-month outperforms one-month estimated IVOL clearly, as one-month coefficient is significantly at -1.708, suggesting that six-months estimation would support large cap examination in future research settings. Ang et al (2009) report similar to Nordic pattern for their U.S. sample period. My sample does not show any significant IVOL coefficients for U.S. data.

5.6.2 Possible shortcomings and future research

Following Birru's (2018) findings regarding Monday-Friday effect, it would make sense to examine further the timing of return recognition for daily return observations for IVOL estimation. Microstructure research in general would offer new views for IVOL estimation. Using daily returns for idiosyncratic volatility estimation over prior one month is most probably subject to multiple microstructure challenges such as bid-ask spread behaviour or asynchronous trading. Therefore, estimation methods should further develop extending the methodology towards microstructure literature. Furthermore, as realised volatility seem to provide additional information on future expected returns, it would make sense to continue examining other than those discussed in my paper, such as exponentially weighted moving variance and similar methods. In addition, implied volatility would open another conversation, bringing in aspects related to its superior information value. To account for implied volatility, one should establish bold requirement for the sample and accept only those stocks which have deep enough derivatives market available. Something which is not widely available for Nordic constituents. Further, since the U.S. sample has on average 11.3% higher monthly idiosyncratic volatilities compared to Nordic, combined with anomalous IVOL findings, it would be justified to examine IVOL in detail under especially high volatile periods. Subsamples provide intuition for more detailed analysis as according to findings IVOL did not perform well during the financial crisis, the era with increased volatility. Respectively, after the crisis volatility levels decreased and IVOL strengthened.

Regarding to quality measure, while control for *quality* succeeds to provide consistent evidence, quality score QMJ itself includes similar elements to IVOL and therefore these two factors possibly explain same variation. QMJ subcomponents *BAB bet-against-beta*-factor (Frazzini & Pedersen, 2014) and *EVOL earnings volatility*, to least, are possibly closely connected to return variation captured by volatility measures used for IVOL construction in this study. Further examination of QMJ subcomponents would be fruitful to providing new evidence on relation between idiosyncratic volatility, quality, and size. Subcomponent specific analysis is actually demonstrated by the authors of QMJ, as they (Asness et al, 2014) use narrow quality factor RMW[13] to develop their broad quality QMJ score. Deriving from that, IVOL and quality components should be challenged against each other and then the best performing components could form revised QMJ. Critics to the authors of QMJ goes as they combine all available and possible profitability measures with numerous other scores and make it as a one single score. It is not even that they do it, but how they do it, the construction does not explicitly account which subcomponents are necessarily required for averaging the final score. Therefore, realised weights and internal binding of QMJ subcomponents may vary. QMJ wide foundation, utilising multiple subcomponents, provides information on future returns within U.S. sample, in which it is originally applied by the authors, but based on that we cannot generalise the result to cover other markets or samples. Which then motivates next research attempts to comprehensively examine the relation between quality, as measured by QMJ, and future stock returns in Nordics.

Also, further to develop the analysis, other multifactor models should be applied to extend the FF-3 as a reference model. However, as my work covers Nordic market, before applying another risk factor to the model, extended model should be validated and fully understood within the scope of this specific market. Dataset question arises as well, as I use Datastream market data and show insignificant IVOL anomaly for U.S., the equal sample period should be evaluated with CRSP data for the sake of consistency. Regarding to size, research typically see investment landscape with binary segmenting and applies two categories such as small or big or possibly adds third segment in the middle. I do not say that the methodology should extend to continuous scale, but possibly bit further on that direction.

[13] RMW refers to profitability factor, one of the factors of five-factor model from Fama and French (2015).

6 Conclusion

In this paper, I examine low-volatility anomaly existence in Nordic aggregated stock market and its constituent markets. Fundamentals of financial theory suggests that higher risk should be compensated with higher return. Theory is not always respected as I show that idiosyncratic low-volatility stocks is observed to outperform stocks with higher idiosyncratic volatility. Paper continues Ang et al (2006, 2009) comprehensive work in which they reveal the effect first in the United States and then continue with multiple international markets. I add to earlier literature that, IVOL anomaly exists in Nordics over examined sample period of 2001/01-2017/12, survives the control for quality, is largest within medium sized and big underpriced stocks, as well as among junk or neutral stocks. In contrast, the anomaly is not observed among portfolios highest in quality or smallest in size. Additional evidence on the United States is provided as a benchmark, as contrary results are compared to earlier literature. I observe no idiosyncratic low-volatility anomaly in the United States for equal period with Nordics examination. However, extending the analysis to cover earlier period of U.S. market, I replicate and confirm the findings of Ang et al (2009) for the period of 1980-2003, equal to their sample period. Idiosyncratic volatility puzzle, or anomaly, is therefore one of those phenomena which is observed in earlier data within U.S. market but has faded out during the recent past.

In the Nordics, low idiosyncratic volatility portfolios outperform high idiosyncratic volatility stocks clearly *post-crisis* while *pre-crisis* and during the *financial crisis* the anomaly lacks consistency. Strongest performance is reported from long-short portfolio among medium sized Nordic stocks with monthly alpha 1.6% (5.19, t-stat) and excess return 1.5% (3.22). Portfolio is long 42 low and short 43 high idiosyncratic volatility stocks, returning 14 euros in the end of sample period from one invested euro in the beginning. Portfolio's mean market equity value at EUR 108 million suggest that only small positions would fit in the strategy, and if so, relaxed timing is mandatory. However, in addition to medium stocks' performance, big Nordic stocks with mean market value at EUR 3 billion, deliver monthly large alpha 1.4% with excess return 1.2%. Provided findings on lagged idiosyncratic volatility predicting future excess returns are good evidence how, possibly behavioural, anomalies succeed to exist post-publication for some markets. Sophisticated investors seem to face sufficient limitations to arbitrage and correct observed mispricing, which then leaves some excess returns available for new harvesters.

7 References

Alquist, R., Israel, R. & Moskowitz T. (2018). Fact, fiction and the size effect. Journal of Portfolio Management, 45(1), 3-30.

Altman, E. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. The Journal of Finance, 23(4), 589-609. doi:10.2307/2978933

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. Journal of financial markets, 5(1), 31-56.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross‐section of volatility and expected returns. The Journal of Finance, 61(1), 259-299.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2009). High idiosyncratic volatility and low returns: International and further US evidence. Journal of Financial Economics, 91(1), 1-23.

Asness, C. S., Frazzini, A., & Pedersen, L. H. (2014). Quality minus junk. Review of Accounting Studies, 1-79.

Asness, C., Frazzini, A., Israel, R., Moskowitz, T. J., & Pedersen, L. H. (2018). Size matters, if you control your junk. Journal of Financial Economics, 129(3), 479-509.

Baker, M., Bradley, B., & Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. Financial Analysts Journal, 67(1), 40-54.

Bali, T. G., & Cakici, N. (2008). Idiosyncratic volatility and the cross section of expected returns. Journal of Financial and Quantitative Analysis, 43(1), 29-58.

Banz, R. W. (1981). The relationship between return and market value of common stocks. Journal of financial economics, 9(1), 3-18.

Birru, J. (2018). Day of the week and the cross-section of returns. Journal of Financial Economics, 130(1), 182-214.

Black, Fischer, Michael C. Jensen, and Myron S. Scholes. 1972. "The Capital Asset Pricing Model: Some Empirical Tests." In Studies in the Theory of Capital Markets, edited by Michael C. Jensen, 79-121. New York: Praeger Publishers Inc.

Carhart, M. M. (1997). "On Persistence in Mutual Fund Performance". The Journal of Finance. 52 (1): 57–82. doi:10.1111/j.1540-6261.1997.tb03808.x # 4-factor??

Chan, L. K., Chen, H. L., & Lakonishok, J. (2002). On mutual fund investment styles. The Review of Financial Studies, 15(5), 1407-1437.

Chordia, T., Subrahmanyam, A., & Tong, Q. (2014). Have capital market anomalies attenuated in the recent era of high liquidity and trading activity?. Journal of Accounting and Economics, 58(1), 41-58.

Daniel, K., & Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns. the Journal of Finance, 52(1), 1-33.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica: Journal of the Econometric Society, 987-1007.

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. the Journal of Finance, 47(2), 427-465.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of financial economics, 33(1), 3-56.

Fama, E. F., & French, K. R. (2008). Dissecting anomalies. The Journal of Finance, 63(4), 1653-1678.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. Journal of financial economics, 116(1), 1-22.

Fama, E. F., & French, K. R. (2017). A five-factor asset pricing model. Journal of financial Economics, 116(1), 1-22.

Fama, E. F., & French, K. R. (2017). International tests of a five-factor asset pricing model. Journal of financial Economics, 123(3), 441-463.

Fama, E. F., & French, K. R. (2018). Choosing factors. Journal of Financial Economics, 128(2), 234-252.

Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. Journal of political economy, 81(3), 607-636.

Fama, E.F., French, K.R., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427–465.

Fink, J., K. Fink, and H. He (2012). Expected Idiosyncratic Volatility Measures and Expected Returns. Financial Management 41(3), 519–553.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. Journal of Financial Economics, 111(1), 1-25.

Fu, F. (2009). Idiosyncratic risk and the cross-section of expected stock returns. Journal of financial Economics, 91(1), 24-37.

Hou, K., & Loh, R. K. (2016). Have we solved the idiosyncratic volatility puzzle?. Journal of Financial Economics, 121(1), 167-194.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. Journal of Finance 48, 65–92.

Jin, Lucy, 2013, Idiosyncratic volatility, arbitrage risk, and anomaly returns, PhD thesis, University of Pennsylvania.

Johnson, T., 2004. Forecast dispersion and the cross section of expected returns. Journal of Finance 59, 1957–1978.

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. The journal of finance, 20(4), 587-615.

Malkiel, B. G., & Xu, Y. (2002). Idiosyncratic risk and security returns. University of Texas at Dallas (November 2002).

Markowitz, H. (1952). Portfolio selection. The journal of finance, 7(1), 77-91.

Merton, R.C., 1987. Presidential address: a simple model of capital market equilibrium with incomplete information. Journal of Finance 42, 483–510.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. The Journal of finance, 32(4), 1151-1168.

Newey, W.K., West, K.D., 1987. A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.

Ohlson, J. A. (1980). Financial ratios and the probabilistic prediction of bankruptcy. Journal of accounting research, 109-131.

Ross, Stephen (1976). "The arbitrage theory of capital asset pricing". Journal of Economic Theory. 13 (3): 341–360. doi:10.1016/0022-0531(76)90046-6.

Shanken, J. (1992). On the estimation of beta-pricing models. The review of financial studies, 5(1), 1-33.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), 425-442.

Shumway, T. (1997). The delisting bias in CRSP data. The Journal of Finance, 52(1), 327-340.

Spiegel, M. I., & Wang, X. (2005). Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk. Yale ICF Working Paper No. 05-13; EFA 2005 Moscow Meetings Paper.

Stambaugh, R. F., Yu, J., & Yuan, Y. (2015). Arbitrage asymmetry and the idiosyncratic volatility puzzle. The Journal of Finance, 70(5), 1903–1948.

Appendix

A1.

Extension: cross-sectional and time series analysis of U.S. sample on the period from 1980 to 2003

Ang et al. (2009) reported significant findings on IVOL, proposing that past idiosyncratic volatility predicts future returns in the United States but also among multiple developed markets. Similar findings for U.S. market are reported by Hou and Loh (2016) for the period of 1963-2012. Both Ang et al and Hou and Loh use CRSP research data as their data source for U.S. I find similar effect in Nordics for 2001-2017 but for the U.S., I report insignificant IVOL for the period which is contrary to earlier findings from Ang et al and Hou and Loh. Therefore, primarily I provide this extension as a proof of consistency in methodology implementation, and secondarily as a preliminary comparison between two different datasets. Whereas CRSP offers comprehensively prepared dataset for research purposes, I use raw market data from Datastream and apply basic cleaning methods, described in the section 4.1, to handle possible errors and outliers. By using Datastream data, I can maintain the consistency between Nordic analysis and U.S. benchmark.

Table A1 describes results from Fama Macbeth cross-section computed for the thesis, and in addition represents coefficients from Ang at al. (2009) as a comparison. First four columns report the results from computation used in this paper while the last column includes coefficients from the paper of Ang et al. Equal time period and methodology are used for the analysis for both. Comparing the coefficients, noticeable similar patterns are observed, even though Ang et al use CRSP dataset while my analysis is based on data fetched from Datastream. IVOL coefficients are especially similar in value-weighted WLS regression settings, Ang et al providing higher significance with larger coefficient with 5441 average number of stocks in their sample. Sample from Datastream includes less than half of that, mean number of 2200 stocks, after proposing requirement for market equity value of the firm minimum at USD 5 million. Respecting the fact that the datasets from CRSP and Datastream deviates from each other, the findings are apparently in similar direction and therefore the results rationalises and proofs the methodology implementation used in the analysis for this thesis work. Also, as it confirms successful replication of Ang et al (2009) findings on past idiosyncratic volatility predicting future stock returns.

Table A1. Extended U.S. sample: FMB regressions 1980-2003

Table reports Fama Macbeth (1973) OLS and WLS regressions for the United States over the sample period of Ang et al (2009) from January 1980 to December 2003. Numbers are expressed in USD. First four columns represent results from computations for the thesis, last column reports results from Ang et al (2009) as a comparison. Similar methods have been applied. LHS variable, monthly excess returns of a firm, is regressed on constant, idiosyncratic volatility *IVOL* computed from multiple previous time periods using daily returns, contemporaneous factor loadings *βMKT*, *βSMB*, *βHML* with respect to FF3 returns from K. French database, and firm characteristics in the beginning of month. *Size* is log market equity value of the firm in the beginning of the month, *BE/ME* is book-to-market for the firm available six months prior. *RET lagged* is the stocks return over preceding six months. *Adj R2* is time series average of adjusted R^2s from cross sectional regressions. *N* denotes mean number of constituent stocks over full sample period. Newey-West robust t-statistics with four lags are reported in square brackets. Panel B reports economic effect of moving from 25th to 75th volatility percentile in monthly terms.

	IVOL estimated from daily returns, 1980-2003				
	OLS	WLS	WLS	WLS	WLS
	1 month	1 month	6 months	12 months	1 month
					Ang et al
Panel A: FMB coefficients					
Constant	2.469***	2.294***	2.480***	2.237***	1.746***
	[6.52]	*[4.43]*	*[4.69]*	*[4.23]*	*[3.83]*
IVOL	-0.556*	-1.137***	-1.850***	-1.385**	-2.014***
	[-1.92]	*[-2.77]*	*[-3.45]*	*[-2.38]*	*[-6.67]*
β MKT	0.436***	0.338	0.455	0.453	0.376***
	[2.79]	*[1.63]*	*[1.59]*	*[1.54]*	*[4.52]*
β SMB	-0.134**	-0.280***	-0.201	-0.178	-0.049
	[-2.38]	*[-3.50]*	*[-1.54]*	*[-1.14]*	*[-1.19]*
β HML	-0.107	-0.036	-0.076	-0.024	-0.051
	[-1.34]	*[-0.33]*	*[-0.44]*	*[-0.13]*	*[-1.69]*
Size	-0.240***	-0.147***	-0.165***	-0.141***	-0.157***
	[-5.20]	*[-2.99]*	*[-3.56]*	*[-3.03]*	*[-3.14]*
BE/ME	0.349***	0.205	0.277**	0.224	0.282***
	[4.32]	*[1.59]*	*[2.32]*	*[1.61]*	*[3.87]*
RET lagged	0.295	0.167	0.637*	0.210	-0.001
	[1.19]	*[0.47]*	*[1.89]*	*[0.60]*	*[0.28]*
Adj R2	0.080	0.165	0.198	0.189	0.046
N	2227	2227	2212	2101	5441
Panel B: Idiosyncratic volatility percentiles and economic effect					
25th pctl	27.04	27.04	32.60	33.52	25.01
75th pctl	59.94	59.94	66.70	66.86	61.10
Economic effect (%)	-0.18	-0.37	-0.63	-0.46	-0.73
Panel C: Descriptive summary, time-series averages					
ERET	1.67	1.67	1.66	1.65	
ME bn	1.67	1.67	1.68	1.75	0.98
ME 50th bn	0.22	0.22	0.23	0.24	
BE/ME	0.77	0.77	0.77	0.76	0.81

Significance: *, **, *** refers to p<0.1, p<0.05, p<0.01 respectively

A2.

Table A2. FMB correlations
Table reports correlations between variables used for Fama Macbeth (1973) regression setting for full sample period of 2001/01-2017/12. QMJ quality score is included in Nordic samples.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Pearson correlations, Nordic									
(1)	QMJ	1.00							
(2)	IVOL	-0.16	1.00						
(3)	MKT	0.06	0.15	1.00					
(4)	SMB	-0.15	-0.01	-0.81	1.00				
(5)	HML	0.03	-0.16	-0.58	0.32	1.00			
(6)	Log Size	0.00	0.14	-0.21	0.10	0.23	1.00		
(7)	BE/ME	0.28	-0.03	0.02	-0.06	0.13	0.13	1.00	
(8)	RET lagged	-0.04	-0.19	-0.38	0.23	0.25	0.03	-0.03	1.00

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel B. Spearman rank correlations, Nordic									
(1)	QMJ	1.00							
(2)	IVOL	-0.15	1.00						
(3)	MKT	0.09	0.10	1.00					
(4)	SMB	-0.16	-0.01	-0.76	1.00				
(5)	HML	-0.01	-0.07	-0.39	0.19	1.00			
(6)	Log Size	-0.02	0.21	-0.26	0.17	0.18	1.00		
(7)	BE/ME	0.29	-0.01	0.04	-0.06	0.02	0.16	1.00	
(8)	RET lagged	-0.06	-0.19	-0.18	0.14	0.03	-0.05	-0.09	1.00

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel C. Pearson correlations, United States								
(1)	IVOL	1.00						
(2)	MKT	0.44	1.00					
(3)	SMB	0.06	0.28	1.00				
(4)	HML	-0.17	0.10	0.06	1.00			
(5)	Log Size	-0.07	-0.29	-0.09	0.07	1.00		
(6)	BE/ME	-0.12	0.02	0.03	0.26	-0.03	1.00	
(7)	RET lagged	-0.35	-0.37	-0.03	-0.02	0.19	-0.14	1.00

A3.

Table A3. Descriptive summary, subsamples

Main sample is divided in three subsamples for further analysis. Nordic values are expressed in EUR currency. USD-nominated data from the United States is used for benchmarking and processed equally with the Nordic data. *Number of firms* is the average number of valid firms included in the sample for each moment. *With full coverage* is the number of constituents which are present in all moments in the sample. *Size* describes the statistics of time series market equity.

	Number of firms	With full coverage	Size (m)			
			Mean	25th	50th	75th
Panel A. Sub sample "pre-crisis" 2001/01 - 2007/09 (81 months)						
Nordic	701	460	878	28	85	342
Finland	124	98	1292	37	116	520
Denmark	160	123	603	27	83	266
Norway	154	79	725	33	92	336
Sweden	262	159	863	24	74	329
United States*	3972	2598	3271	104	376	1434
Panel B. Sub sample "financial crisis" 2007/10 - 2009/06 (21 months)						
Nordic	806	680	853	27	81	345
Finland	122	117	1304	42	131	625
Denmark	176	154	674	29	71	268
Norway	193	159	860	39	99	376
Sweden	313	249	708	19	62	264
United States*	3714	3363	3423	100	385	1558
Panel C. Sub sample "post-crisis" 2009/07 - 2017/12 (102 months)						
Nordic	785	464	1251	28	104	540
Finland	120	83	1246	46	181	790
Denmark	139	91	1567	24	78	716
Norway	175	95	1116	38	127	547
Sweden	350	194	1132	24	85	448
United States*	3606	2185	5283	158	695	2748

* *U.S. dollars*

A4.

Table A4. Volatilities, multiple estimation periods, subsamples

Main sample consist of Nordic EUR-nominated data. USD-nominated data from the United States is used for benchmarking and processed equally with the Nordic data. Table describes total volatility (TVOL) and idiosyncratic volatility (IVOL) estimated from daily and monthly returns. All volatility values are annualised by multiplying estimated percentage of volatility with square of 250 for daily volatility and with square of 12 for monthly volatility. 1m to 36m describes prior window used for volatility computation. *TVOL* is a time-series mean of simple standard deviation measured from excess return of each stock. *IVOL* is defined as time-series mean of simple standard deviation measured from variation in error-terms of FF3-factor estimation for each stock.

	Daily data					Monthly data				
	TVOL (%)		IVOL (%)			TVOL (%)		IVOL (%)		
	1m	12m	1m	6m	12m	12m	24m	12m	24m	36m
Sub sample "pre-crisis" 2001/01 - 2007/09 (81 months)										
Nordic	38.97	43.11	33.59	37.99	38.58	38.06	40.29	27.94	31.42	32.50
Finland	33.72	37.59	28.92	33.29	34.35	32.22	34.64	23.72	26.91	28.20
Denmark	29.12	33.17	25.66	30.06	30.57	30.16	31.26	23.24	26.03	26.79
Norway	43.24	48.30	37.00	42.58	43.54	42.14	44.59	30.87	34.50	35.19
Sweden	45.36	49.35	37.43	41.55	42.01	43.84	46.75	29.85	33.91	35.44
United States	42.77	47.57	36.61	41.77	43.21	43.63	46.95	33.07	37.85	40.00
Sub sample "financial crisis" 2007/10 - 2009/06 (21 months)										
Nordic	52.78	49.11	43.66	46.25	42.80	43.11	40.49	28.84	29.78	29.99
Finland	44.50	41.96	36.38	39.36	37.55	34.70	33.11	24.17	25.41	25.08
Denmark	48.30	44.58	41.14	43.80	40.19	38.66	36.63	26.89	27.61	28.29
Norway	56.95	52.48	46.85	50.24	45.42	47.52	43.16	30.07	30.50	31.50
Sweden	55.93	52.42	44.15	46.49	43.60	46.28	44.13	30.14	31.47	31.51
United States	66.47	58.56	51.63	54.58	49.83	47.28	42.41	34.38	33.95	33.29
Sub sample "post-crisis" 2009/07 - 2017/12 (102 months)										
Nordic	40.55	44.97	34.45	38.53	39.18	38.18	40.22	28.36	31.30	31.83
Finland	32.36	35.85	26.67	30.37	31.22	30.62	32.22	21.76	24.41	25.27
Denmark	35.30	39.73	30.37	34.74	35.95	32.85	35.14	25.22	28.70	29.91
Norway	45.55	51.19	38.21	43.43	44.54	42.29	44.74	31.47	34.72	35.25
Sweden	42.80	47.05	35.46	39.10	39.28	40.92	42.93	29.43	32.20	32.56
United States	38.73	44.31	31.61	36.15	37.59	39.52	41.76	29.17	32.80	33.79

A5.

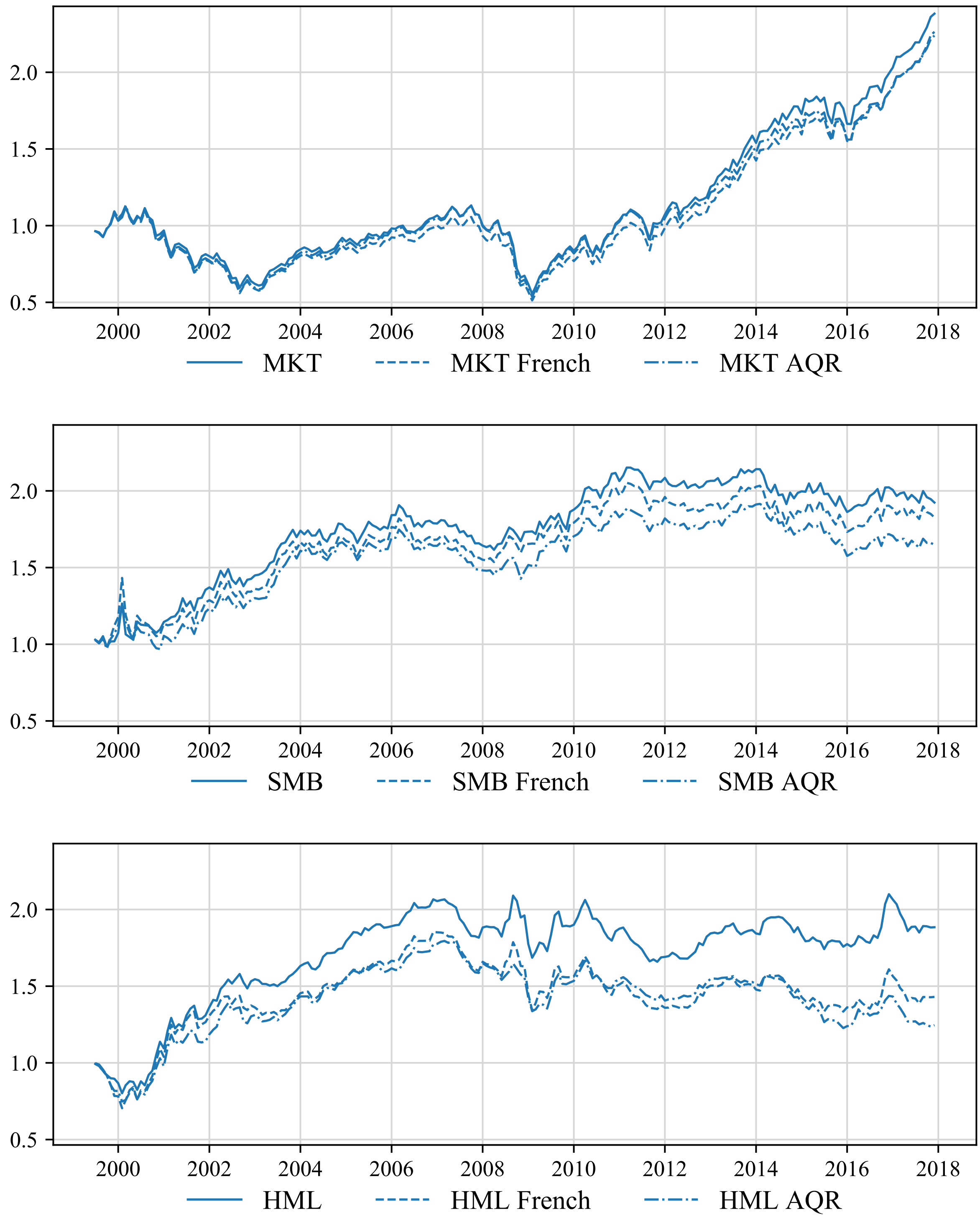

Figure A1. FF-3 portfolio returns in the United States

Plotted are Fama French 3-factor cumulative excess returns, growth of USD 1, computed for U.S. market. Factor construction follows Fama and French (1993), use USD-denominated values. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor. *French* refers to factor returns fetched from Kenneth French website and are illustrated to validate accuracy of data and methods used for the study. *MKT/SMB/HML AQR* refers to factor returns from AQR Capital Management, represented as a benchmark, downloaded from their website. AQR factor construction may vary. On average, self-computed vs. French factor tracking errors are less than what is observed between AQR vs. French.

A6.

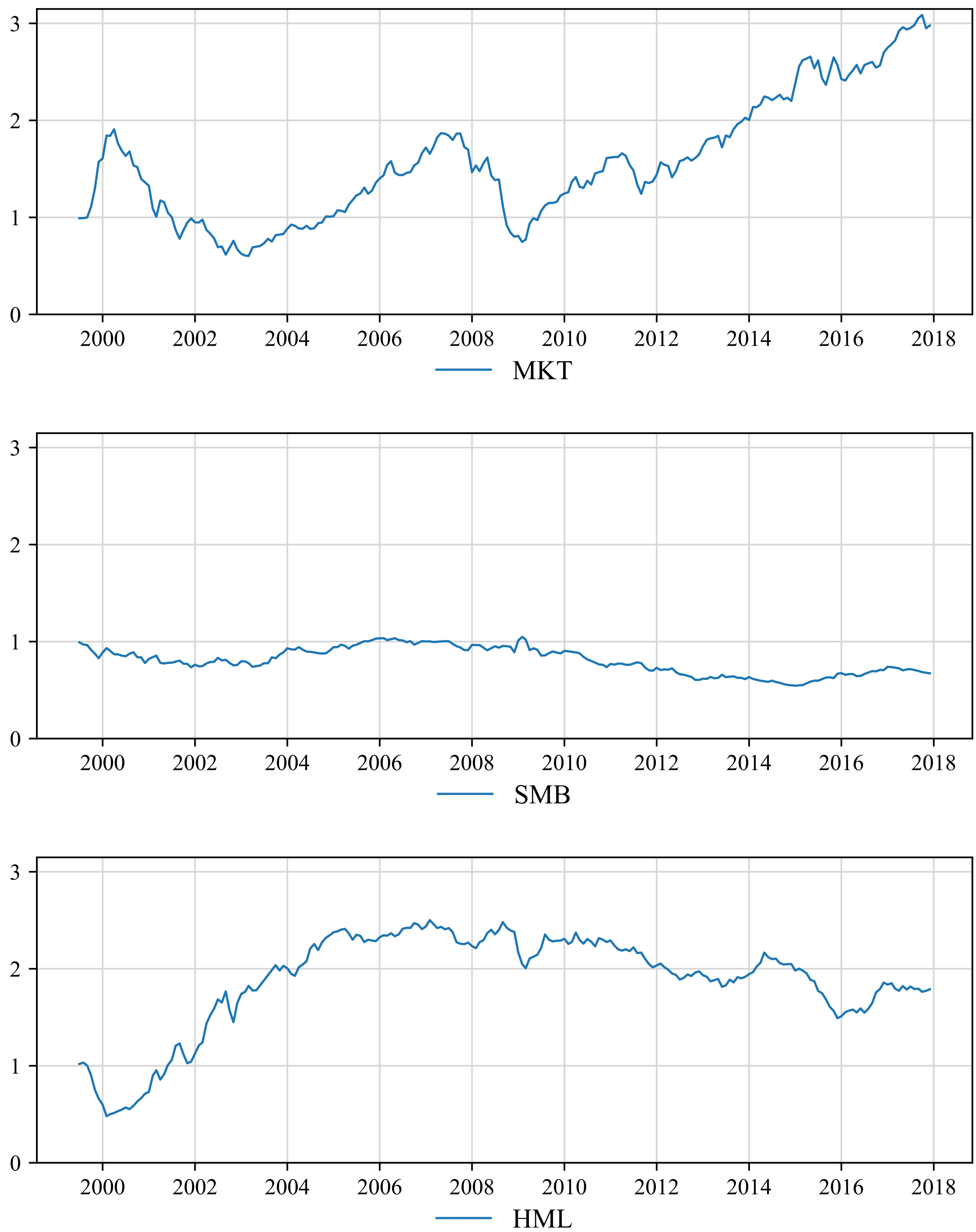

Figure A2. FF-3 portfolio returns in Nordic
Plotted are Fama French 3-factor cumulative excess returns, growth of EUR 1, computed for Nordic aggregated market. Factor construction follows Fama and French (1993), use EUR-denominated values, and are calculated using Nordic stocks listed in main OMX Nordic exchanges. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor.

A7.

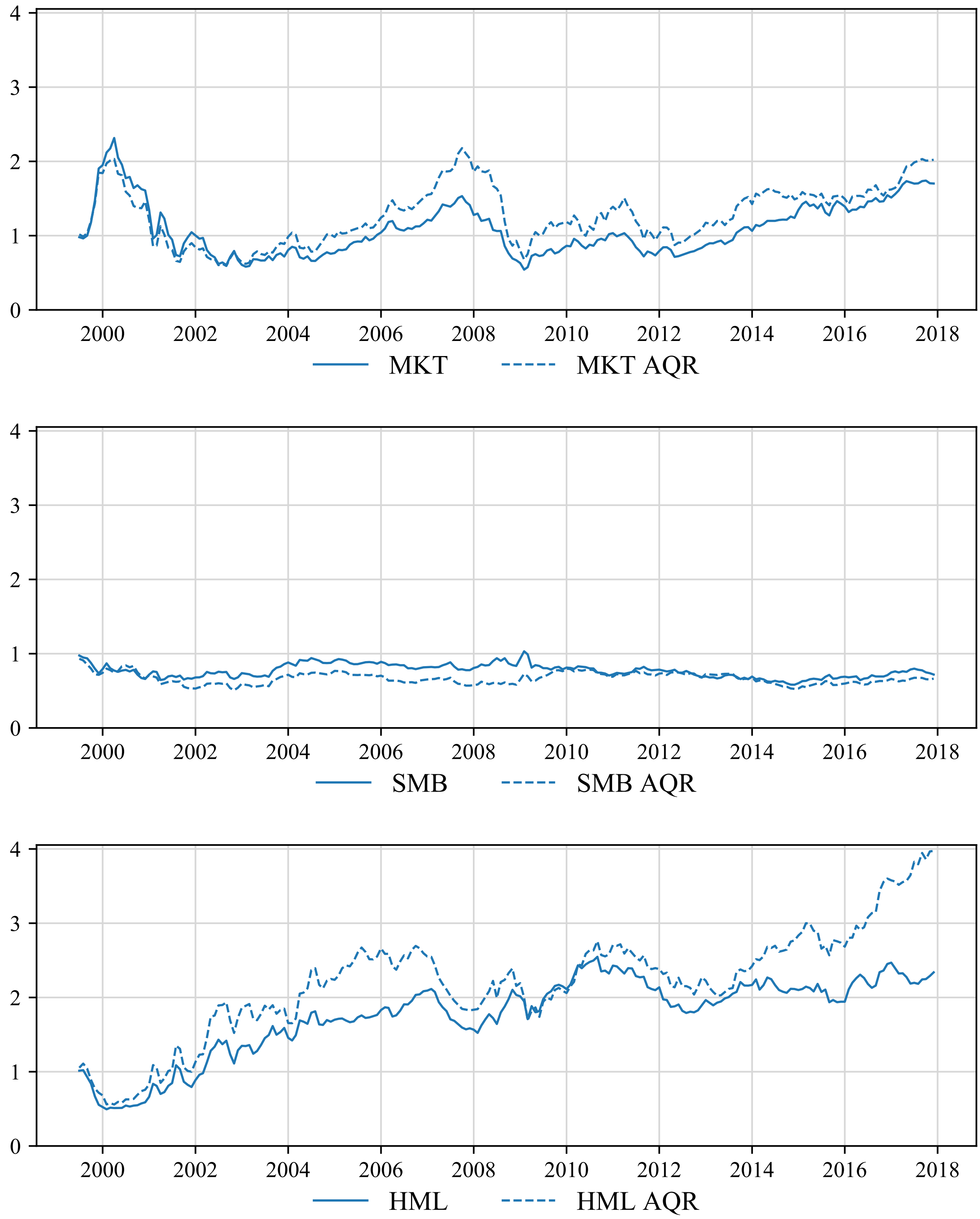

Figure A3. FF-3 portfolio returns in Finland

Plotted are Fama French 3-factor cumulative excess returns, growth of EUR 1, computed for Finnish market. Factor construction follows Fama and French (1993), use EUR-denominated values, and are calculated using Finnish stocks listed in OMX Helsinki. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor. AQR Capital Management publishes USD-denominated factor returns for Nordic markets. *MKT/SMB/HML AQR* refers to factor returns from AQR, represented as a benchmark. Their factor construction may vary. Please see related information from AQR website.

A8.

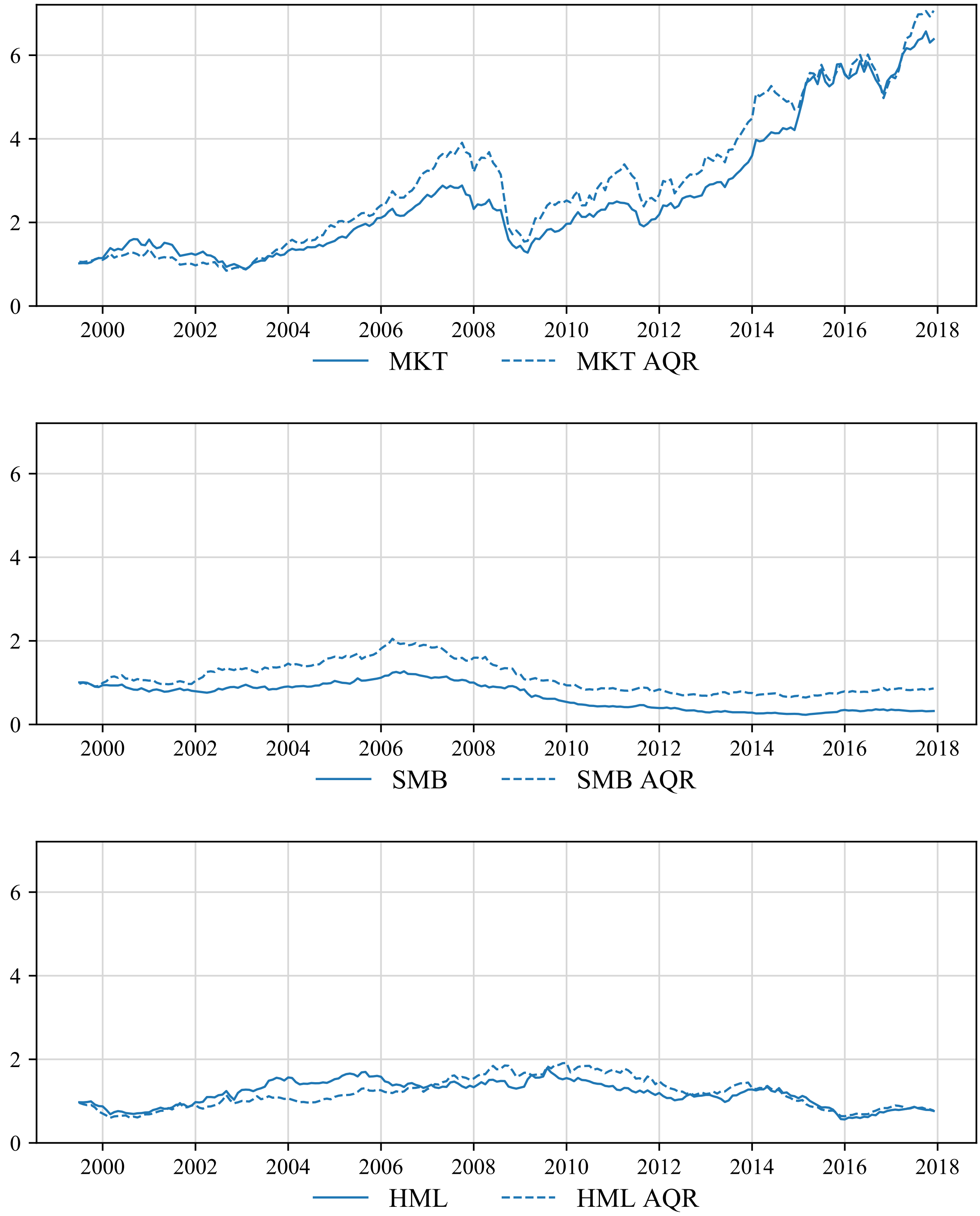

Figure A4. FF-3 portfolio returns in Denmark
Plotted are Fama French 3-factor cumulative excess returns, growth of EUR 1, computed for Danish market. Factor construction follows Fama and French (1993), use EUR-denominated values, and are calculated using Danish stocks listed in OMX Copenhagen. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor. AQR Capital Management publishes USD-denominated factor returns for Nordic markets. *MKT/SMB/HML AQR* refers to factor returns from AQR, represented as a benchmark. Their factor construction may vary. Please see related information from AQR website.

A9.

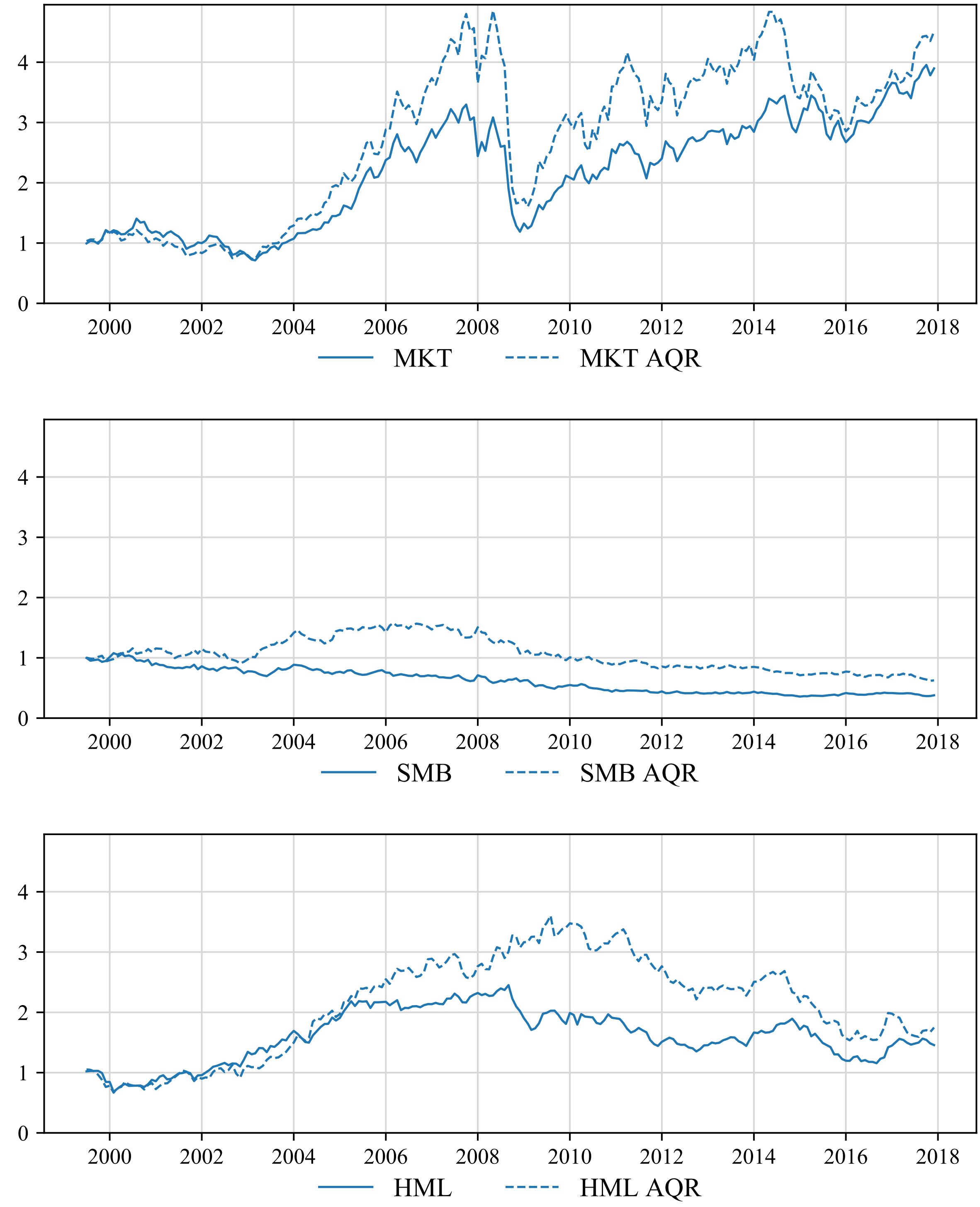

Figure A5. FF-3 portfolio returns in Norway
Plotted are Fama French 3-factor cumulative excess returns, growth of EUR 1, computed for Norwegian market. Factor construction follows Fama and French (1993), use EUR-denominated values, and are calculated using Norwegian stocks listed in Oslo Börs. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor. AQR Capital Management publishes USD-denominated factor returns for Nordic markets. *MKT/SMB/HML AQR* refers to factor returns from AQR, represented as a benchmark. Their factor construction may vary. Please see related information from AQR website.

A10.

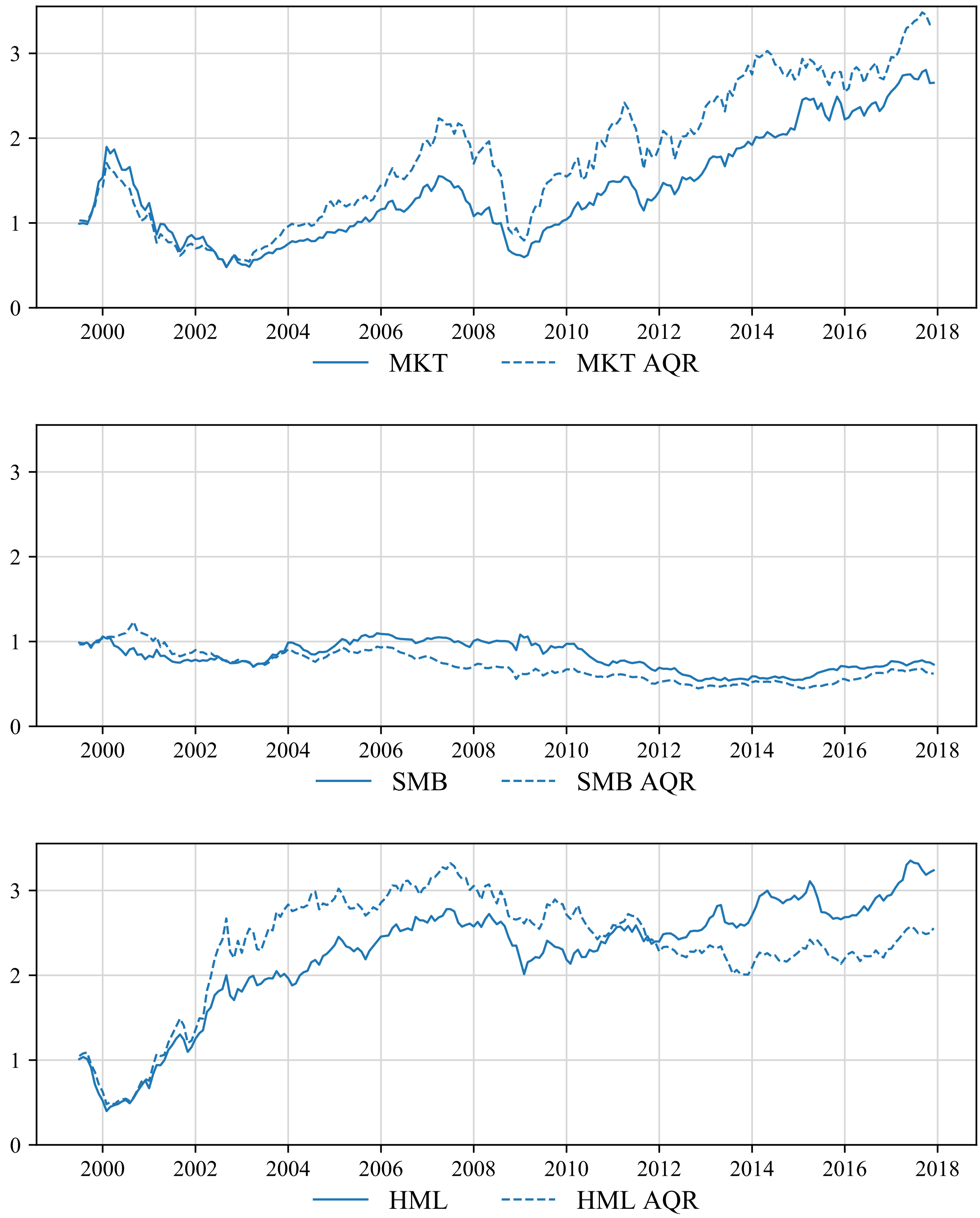

Figure A6. FF-3 portfolio returns in Sweden

Plotted are Fama French 3-factor cumulative excess returns, growth of EUR 1, computed for Swedish market. Factor construction follows Fama and French (1993), use EUR-denominated values, and are calculated using Swedish stocks listed in OMX Stockholm. *MKT* refers to market, *SMB* is small-minus-big, and *HML* is high-minus-low value factor. AQR Capital Management publishes USD-denominated factor returns for Nordic markets. *MKT/SMB/HML AQR* refers to factor returns from AQR, represented as a benchmark. Their factor construction may vary. Please see related information from AQR website.